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OM

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ECE

PM 1(B)

ACE

Signals & Systems

=> Signals and Systems:

Analysis

① Introduction

② L.T.I. systems

Approximation

① Fourier series

Transformation

① C.T.F.T

② L.T.

③ D.T.F.T.

④ Z.T.

⑤ D.F.T.

=> **Books:**

① Openhium & Nawab.

② Haykins & Vanveen - IES

③ S&S by Hsu & Ranjan (TMH).

* **Signal:**

=> Signal is an indication about which some amount of information is conveyed.

=> Randomnes is a information i.e. the things which we are dont know i's signal.

=> The operations that are perform on the signals are:

① Enhancing,

② Extracting

③ Storing

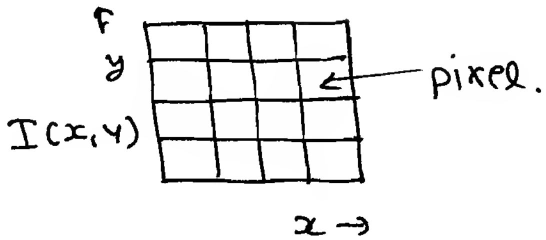
④ Filtering.

☆ Characteristics of a Signal:

⇒ ① More than one independent variable.

e.g. : ① Speech → 1D (time)

② Image → 2D



③ T.V. picture → 3D

$I(x, y, t)$.

⇒ ② Randomness:

⇒ More the Randomness more the information.

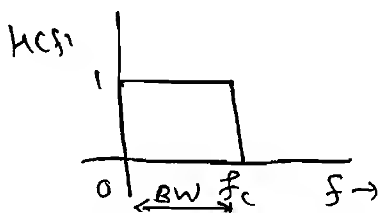
$$I = \log_2 \frac{1}{P_i} = -\log_2 P_i$$

$\therefore$ $P_i = 1/8$ ⇒ $I = 3$ bits

more randm. → $P_i = \frac{1}{32}$ ⇒ $I = 5$ bits.

⇒ ③ Bandwidth:

⇒ In order to know the BW of the channel we should know the BW of signal that's why the BW is one of the chara. of signal.



$BW = f_c$

☆ Types of signal:

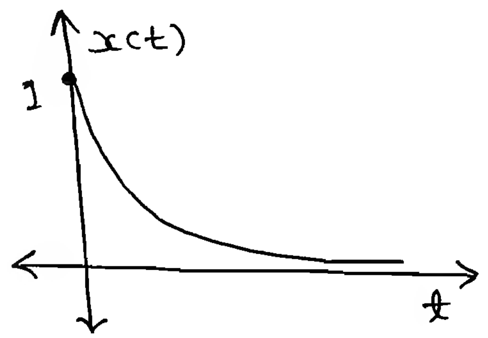
① Continuous time signal:

⇒ A signal which occurs for continuum value of time. is called continuous signal.

⇒ At any instant the amplitude of the signal is known.

⇒ Continuous both in time and amp. is Continuous signal.

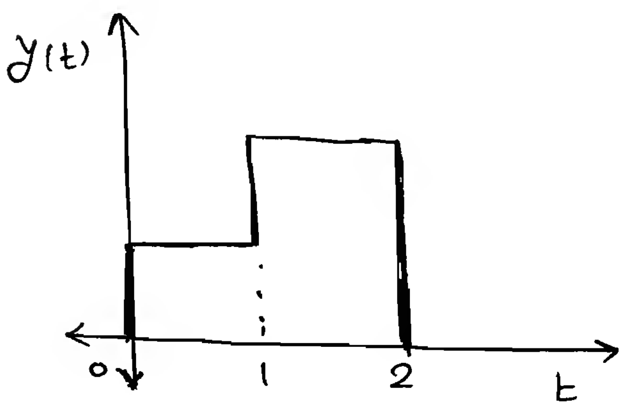
e.g. $x(t) = e^{-3t} \cdot u(t)$



Amp	Time	Signal Type
C	C	Conti. signal
C	D	Discrete signal
D	C	Quantizer
D	D	Digital signal

⇒ Whenever there is sudden changes in the ~~signal~~ level (or) amp is called Discontinuous signal.

e.g. $y(t)$ signal.



at $t = 0, 1, 2$ there are sudden change in amp. so, at $t = 0, 1, 2$, $y(t)$ the amp. is not defined i.e. discontinuous

piece-wise | Discontinuous

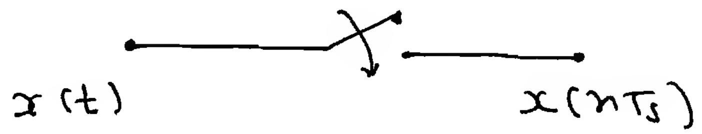
② Discrete signal :-

⇒ A signal which is continuous in amplitude but discrete in time (integer values of time index). is called discrete signal.

⇒ BW will be less than continuous signal.

⇒ Used in concept of Multiplexing which is very easy.

⇒ Sampling:

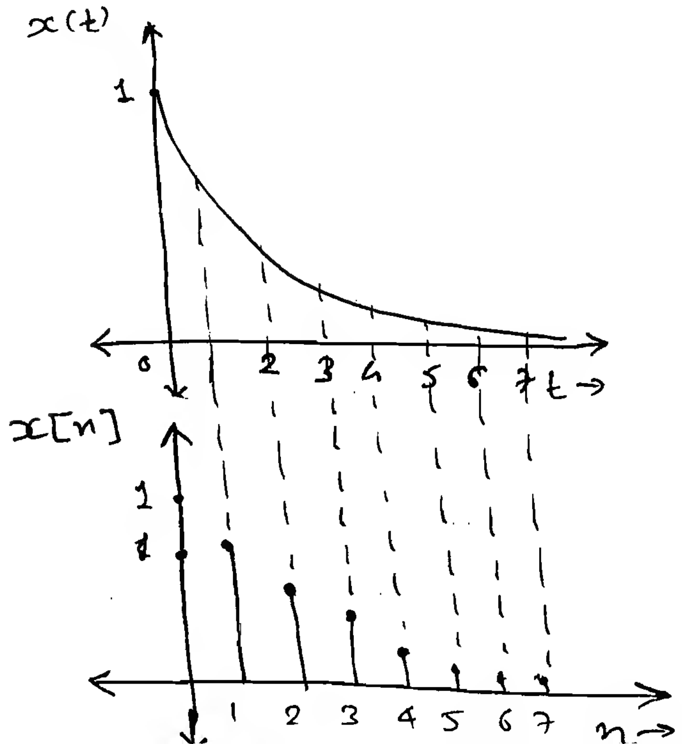


$t = nT_s \quad , \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots$

$t = nT_s$

$T_s =$ Sampling time.

⇒ $x(t) = e^{-3t} . u(t)$.



$x[nT_s] = e^{-3nT_s} . u[nT_s]$

Let, $T_s = 1$ sec

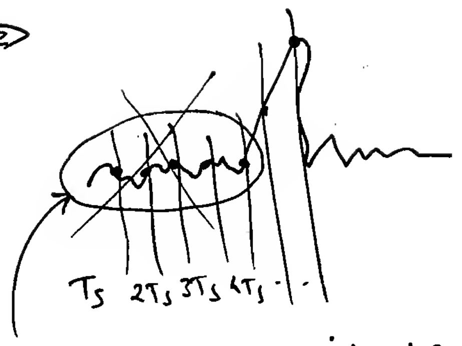
$x[n] = e^{-3n} . u[n]$.

$x[nT_s] \rightarrow x[n]$

for simplicity

$x[nT_s - T_s] \rightarrow x[n-1]$

T_s is depend on samp. theorem. i.e. $f_s \geq 2 f_m$.

⇒ 

$x[nT_s] \rightarrow x[n]$ for simplicity.

So, we can't use $T_s = 1$ sec every time.

Almost amplitude are same for all the sample.

⇒ The purpose of Sampling is multiplexing.

⇒ Every discrete signal is not the Sampled version of the Continuous signal. Some of the discrete signals are Predefine.

$\boxed{\text{NOTE:}}$

⇒ Amp. Conversion from C→D ⇒ Quantizer

⇒ Time Conversion from C→D ⇒ Sampling.

③ Digital Signal:

⇒ A Signal which discrete in time and amplitude (quantized amp.) is Called Digital signal.

⇒ Quantizer is used to Convert the Continuous amp. signal into discrete amp. signal.

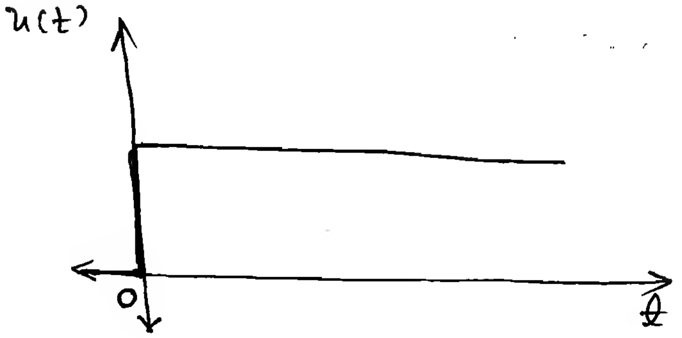
⇒ Sampler is used to Convert Continuous time signal into discrete time signal.

* Some Standard Continuous signal :-

① **Unit Step Function:**

$\Rightarrow$ $u(t) = 1, \quad t > 0$

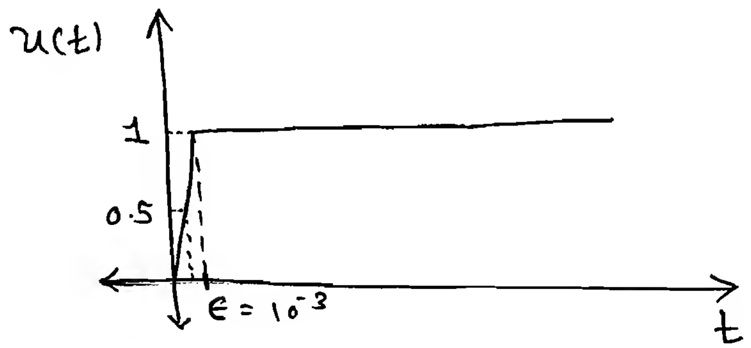
$\quad\quad = 0, \quad t < 0$



$\Rightarrow$ At $t=0$ there is a discontinuity i.e. why are not defining $u(t)$ at $t=0$.

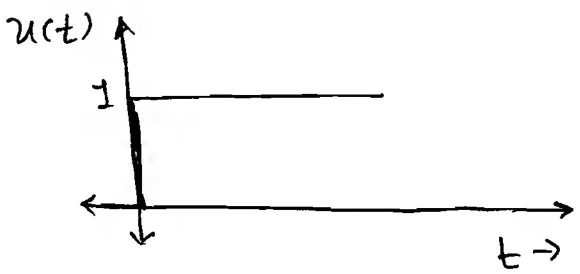
Transient & Bounded amp.

$\Rightarrow$ But practically $u(t)$ signal is as follow in Lab. :

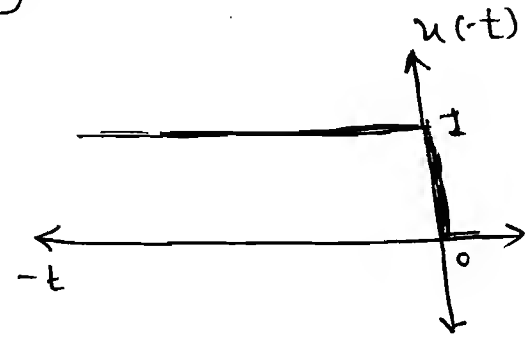


$\Rightarrow$ Mathamatically $u(0) = \frac{1}{2}$.

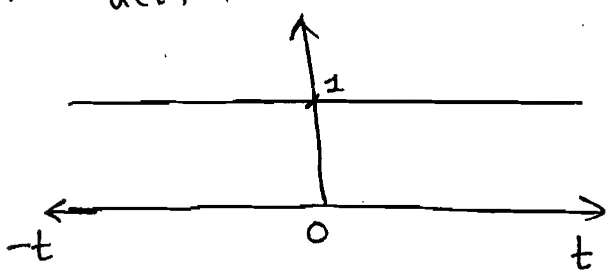
① $u(t)$



② $u(-t)$.



$\Rightarrow$ $u(t) + u(-t)$



$u(t) + u(-t) = 1$ at $t = 0$.

$\therefore u(0) + u(0) = 1$

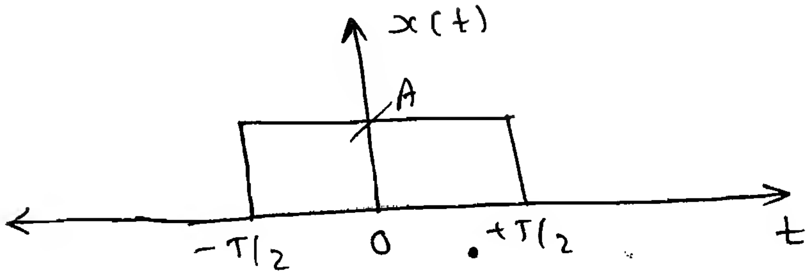
$\Rightarrow u(0) = \frac{1}{2}$.

② **Rectangular function :-**

⇒ $A\,\mathrm{rect}\,(t/T)$ (or) $A\,\Pi\,(t/T)$.

⇒ $A\,\mathrm{rect}\,(t/T) = A \;;\; -\frac{T}{2} < t < T/2$

$= 0 \;;\;$ elsewhere

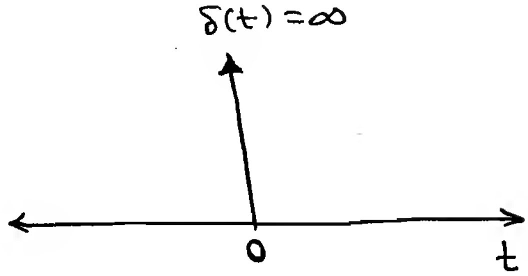


③ **Continuous impulse function** (or) **Dirac delta function:**

⇒ $\delta(t)$.

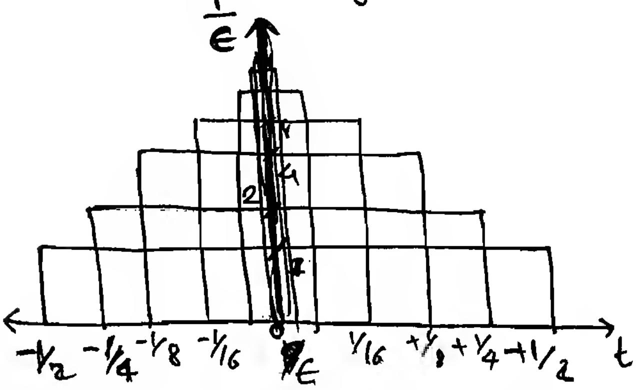
⇒ $\delta(t) = \infty \;;\; t = 0 \;\; (t \to 0)$

$= 0 \;;\; t \neq 0$



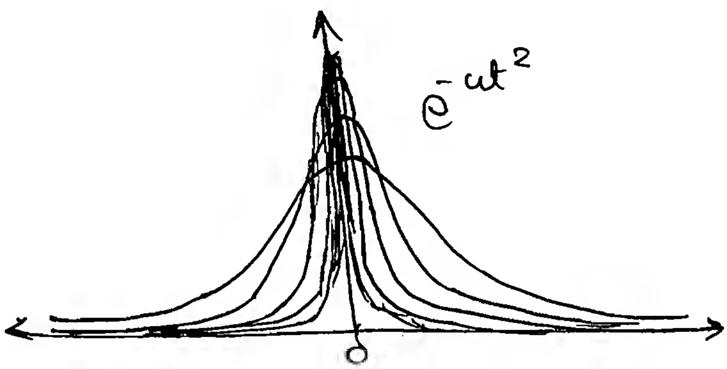
⇒ Used as approximation of standard signals.

⇒ By approximate unit area rectangular f^n we can generate unit impulse f^n.



$\boxed{\epsilon \to 0 \Rightarrow 1/\epsilon \to \infty}$

By Rectangular f^n



By Gaussian f^n.

$\Rightarrow$ $\cancel{\delta(t) \neq 3t}$ ~~kis~~ $x(t) = 3\,\delta(t)$.

i.e. amp. $\neq 3$ but amp. $= \infty$.

but $\boxed{\text{area under } \delta(t) = 3.}$

* **Properties of $\delta(t)$ fn:**

(i) $\displaystyle\int_{-\infty}^{+\infty} \delta(t)\,dt = 1.$

Area Concept are used in Continuous δ fn. (or) cont. impulse fn.

(ii) $\delta(\alpha t) = \dfrac{1}{|\alpha|} \cdot \delta(t).$

$\alpha \to$ Scaling factor.

(iii) Product (or) Sampling: IMP

$\rightarrow$ $x(t) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)$ if $x(t)$ is contn at $t = t_0$.

$t_0 \to$ time shift.

e.g. ① $\cos t \cdot \delta(t - \pi)$. $\Rightarrow t_0 = \pi$

$= \cos \pi$

$= -1.$

② $t \cdot \delta(t) = t_0 \cdot \delta(t)$

as $t_0 = 0$

$= 0 \cdot \delta(t)$

$\boxed{t \cdot \delta(t) = 0}$

(iv) $\delta(t) = \delta(-t).$

(V) Shifting:

$$\int_{t_1}^{t_2} x(t).\ \delta(t-t_0)\, dt = x(t_0) \ ; \quad t_1 \leq t_0 \leq t_2$$
$$= 0 \ ; \quad \text{elsewhere.}$$

Shift should be lies within the limit.

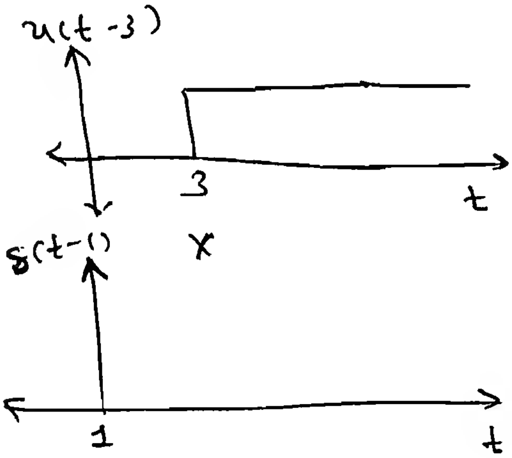
e.g. ① $\displaystyle\int_0^{\infty} (t + \cos\pi t)\ \delta(t-1)\ dt$

Soln: here, $t_0 = 1$.

$= 1 + \cos\pi = 1 - 1 = 0.$

② $\displaystyle\int_0^{\infty} \cos t .\ u(t-3)\ .\ \delta(t-1).\ dt = 0.$

Soln:



no overlap.

So, Ans is 0.

$u(t-3) . \delta(t-1) = 0.$

$= 0$

③ $\displaystyle\int_{-\infty}^{\infty} x(2-t).\ \delta(4-t)\ dt.$

$\delta(4-t) = \delta(t-4) \quad [\because \delta(t) = \delta(-t)].$

So, $t_0 = 4$.

Soln:

$= x(2-t_0).$

$= x(2-4)$

$= x(-2).$

④ $\displaystyle\int_0^{\infty} e^{(t-3)} \cdot \delta(3t-9) \cdot dt$

Sol^n: $\delta(3t-9) = \delta[3(t-3)] = \frac{1}{3}\,\delta(t-3).$

So, $t_0 = 3$

$$= \frac{1}{3} \cdot e^{(t_0-3)}$$

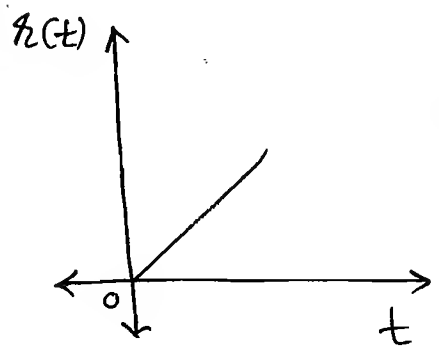
$$= \frac{1}{3} \cdot e^{(3-3)}$$

$$= \frac{1}{3}$$ //

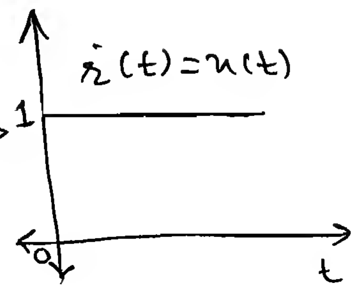
③ Unit Ramp :

$\Rightarrow$ $r(t) = t \;;\; t > 0$

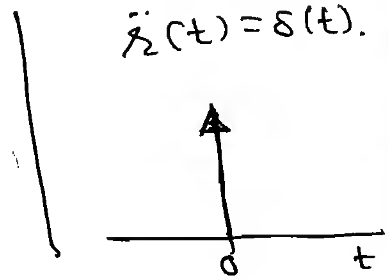
$ = 0 \;;\; t < 0$



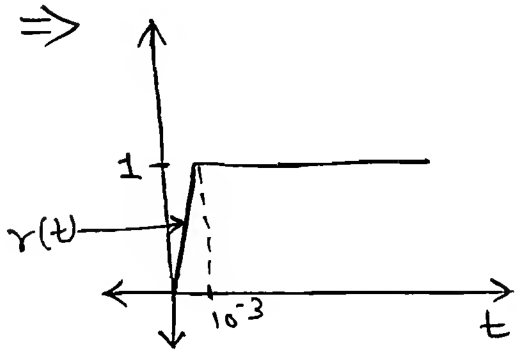
Diff $\Longrightarrow$



Diff $\Longrightarrow$

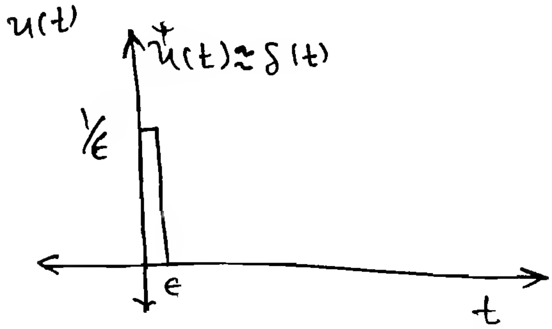


$\Rightarrow$



$$\delta(t) = \frac{d}{dt}\,u(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(t)\,dt.$$

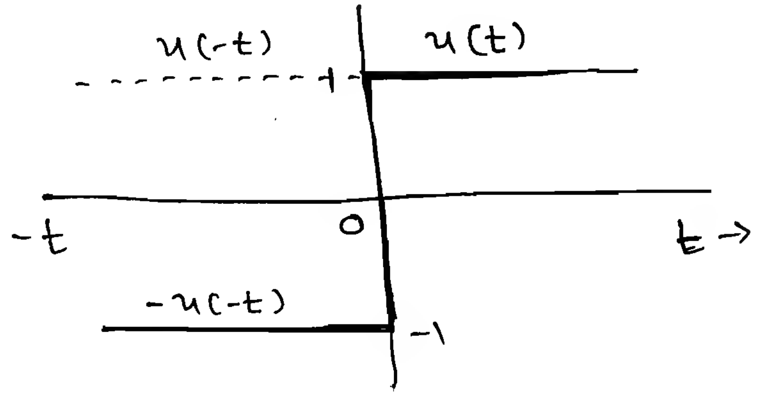


⇒ The function which do not posses higher derivative are singularity f^n.

e.g. $\delta(t)$ & $u(t)$.

④ Signum function:

⇒



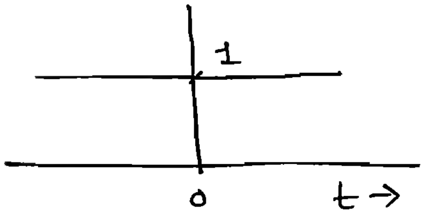
⇒ For any odd f^n amplitude is zero at origin.

$$\text{sign}(t) = u(t) - u(-t)$$

$$\text{sign}(t) = 2u(t) - 1 .$$

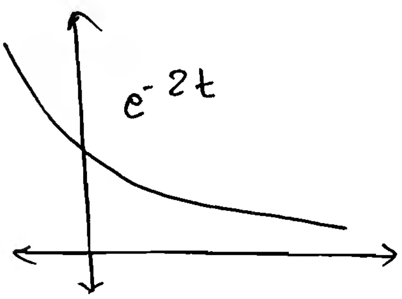
Put $u(-t)$.

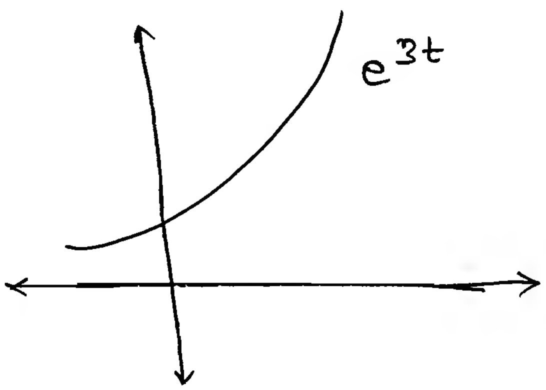
$$u(t) + u(-t) = 1$$ for $t=0$. only.



⑤ Exponentials:

① Real exp. $e^{(\sigma t)}$:





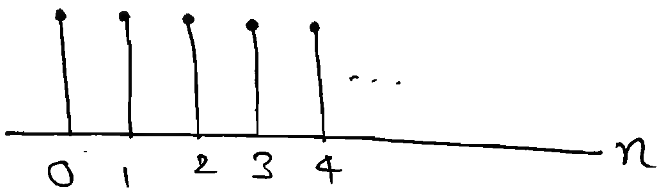
② Complex sinusoid $e^{\pm j\omega_0 t}$.

③ Complex exp.

$$e^{st} = e^{(\sigma + j\omega)t}.$$

⑥ **Unit Step Sequence:**

⇒



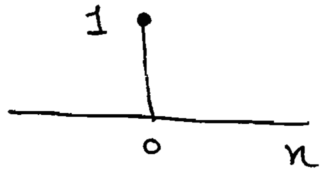
$$u[n] = 1 \; ; \; n \geq 0$$
$$= 0 \; ; \; n < 0.$$

⇒ It is not a Sample version of $u(t)$. because $u(t)$ = discontinuous at $t=0$. but $\underline{u[n] = 1}$ at $\underline{n=0}$.

✓

⑦ **Discrete Impulse:** (or) Kronecker delta:

⇒



$$\delta[n] = 1 \; ; \; n = 0.$$
$$= 0 \; ; \; n \neq 0.$$

$\delta(t)$

⇒ $\boxed{\delta(t) = \frac{d}{dt} u(t).}$

$\boxed{u(t) = \int_{-\infty}^{t} \delta(\tau).d\tau.}$

→ Differentiation ⇒ Difference

t n

$$\nabla x_n = x_n - x_{n-1}.$$

$$\boxed{\delta[n] = u[n] - u[n-1]}$$

⇒ Integration ⇒ Summation.

t n

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

put $n-k=m$.

$$u[n] = \sum_{m=\infty}^{0} \delta[n-m]$$

$$\boxed{u[n] = \sum_{m=0}^{\infty} \delta[n-m].}$$

$\Rightarrow$

$$\boxed{\delta[kn] = \delta[n].}$$

e.g.:- $\delta[4n] = \begin{cases} 1 ; & 4n = 0 \Rightarrow n = 0. \\ 0 ; & \end{cases}$

☆ Transformation of a Signal:-

① **Time - Scaling:-**

$\Rightarrow$ $x(t) \longrightarrow x(at)$

$x[n] \longrightarrow x[mn].$

Used in recording.

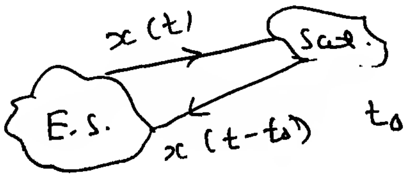
$x(t) = K$ r.p.m. (60 min).

$x(2t) = K$ r.p.m. (30 min).

② **Time - Shift:**

$\Rightarrow$ $x(t - t_0) \mid x[n - n_0]$

Propogation delay.



t_0 = P.D.

③ **Time - reversal:**

$\Rightarrow$ $x[-t] \mid x[-n].$

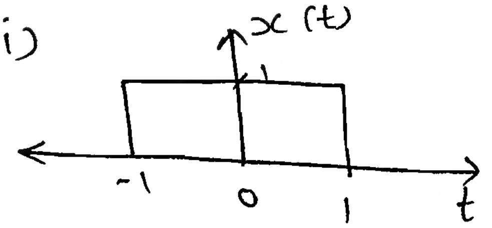
$\Rightarrow$ All ready data are stored. we want to access it one more time we use time reversal. i.e. if we are hearing a music song. and after completion at first time, inorder to re hear it we ~~take~~ rewind it. it is nothing but the time - reversal.

④ **Amplitude Scaling:**

$\Rightarrow$ $k\,x(t) \mid K\,x[n].$

① Time - Scaling:

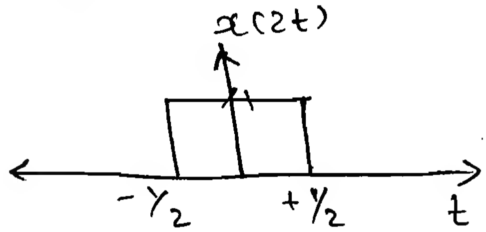
⇒ (i)



$x(t) = 1 \; ; \; -1 < t < 1$

$= 0 \; ; \;$ elsewher

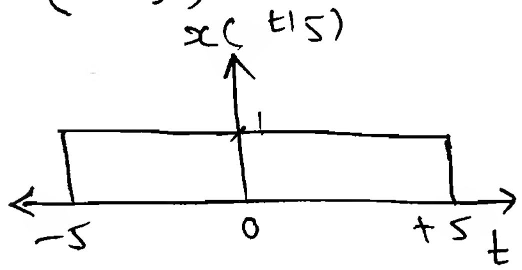
⇒ $x(2t)$



$x(2t) = 1 \; ; \; -1 < 2t < 1$

$x(2t) = 1 \; ; \; -\frac{1}{2} < t < \frac{1}{2}$

⇒ $x(t/5)$



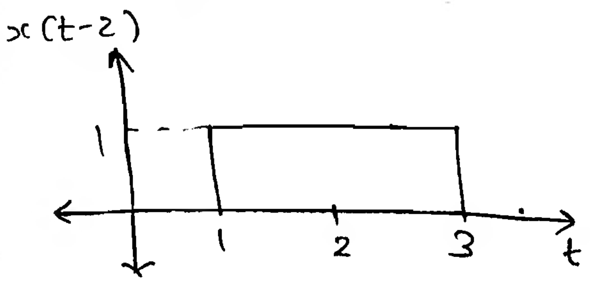
$x(t/5) = 1 \; ; \; -5 < t < 5.$

⇒

$a > 1$	⇒	Compression of $x(t)$.
$a < 1$	⇒	Expam ssion of $x(t)$.

② Time - Shifting:

⇒ (i) $x(t-2)$.

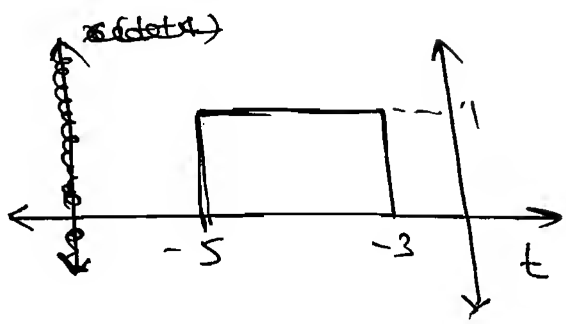


$\boxed{t_0 > 0}$

→ Right Shift

→ Time delayed

(ii) $x(t+4)$



$\boxed{t_0 < 0}$

→ Left Shift

→ Time advance

(olb- line).

⇒ In Real-time, advance (time) is not possible.

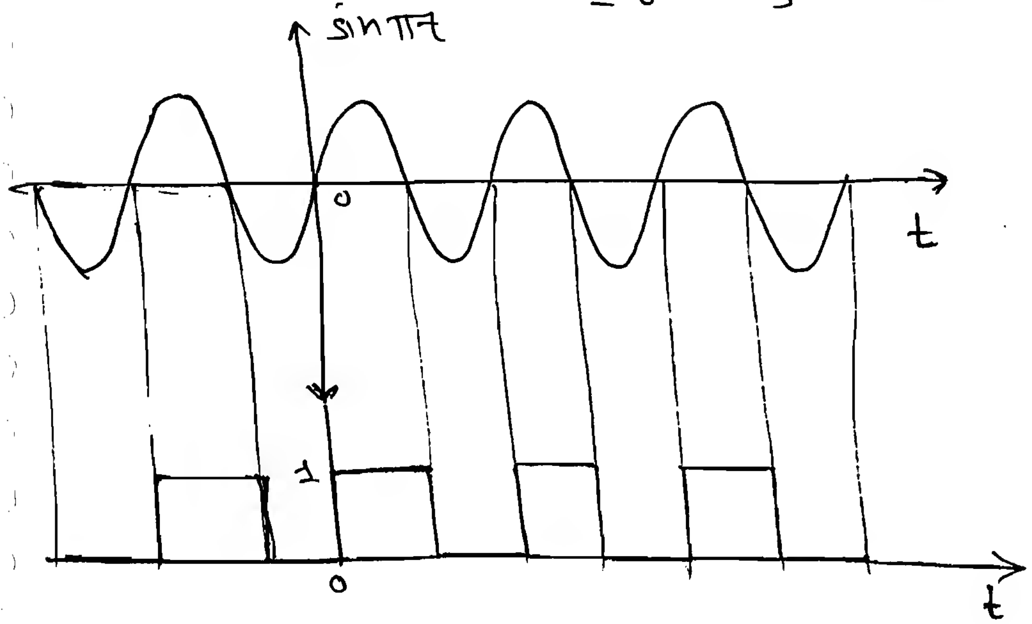
i.e. we can not predict any output before giving a input.

$\boxed{Q-1}$ Draw the following signals:

① $u[\sin\pi t]$.

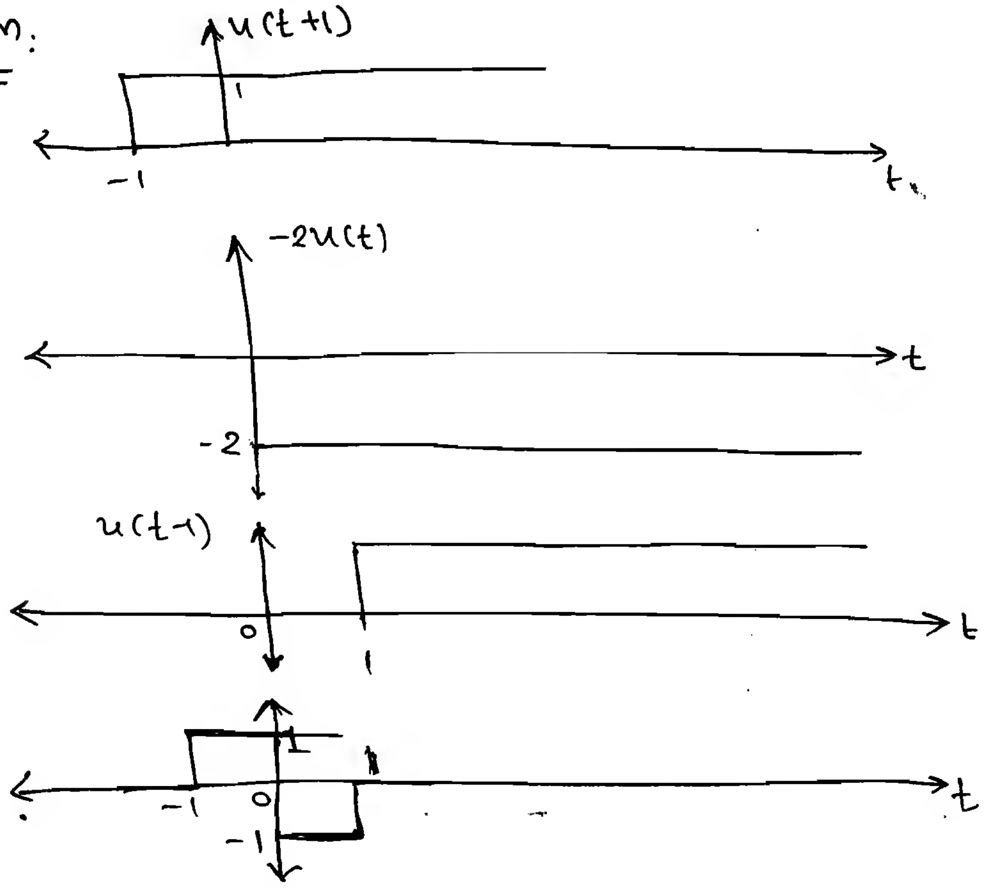
Soln: $u[\sin\pi t] = 1 \quad ; \quad \sin\pi t > 0$.

$\qquad\qquad\quad = 0 \quad ; \quad \sin\pi t < 0$.



② $x(t) = u(t+1) - 2u(t) + u(t-1)$.

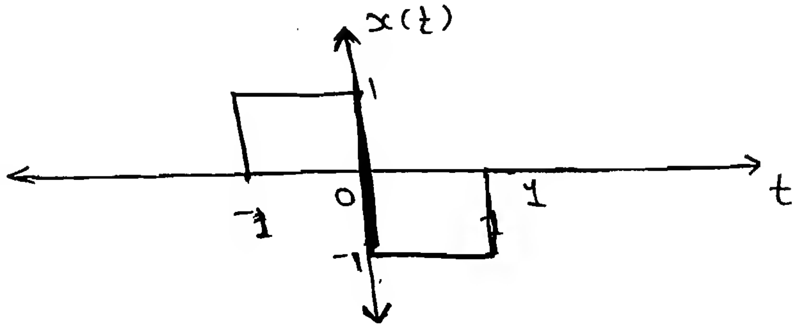
Soln:



(or)

$\Rightarrow \quad x(t) = 1.u(t+1) \quad - 2u(t) \quad + 1u(t-1).$

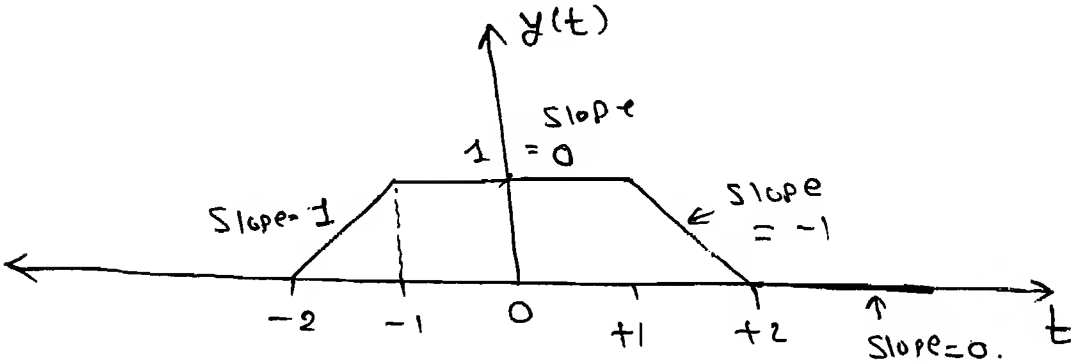
$\quad\quad t = -1 \quad\quad 0 \quad\quad 1$



② $y(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2).$

Sol^n: $y(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2).$

$\quad\quad -2 \quad\quad -1 \quad\quad +1 \quad\quad +2.$



$\Rightarrow$

$-\infty < t < -2 \Rightarrow$ Slope = 0.

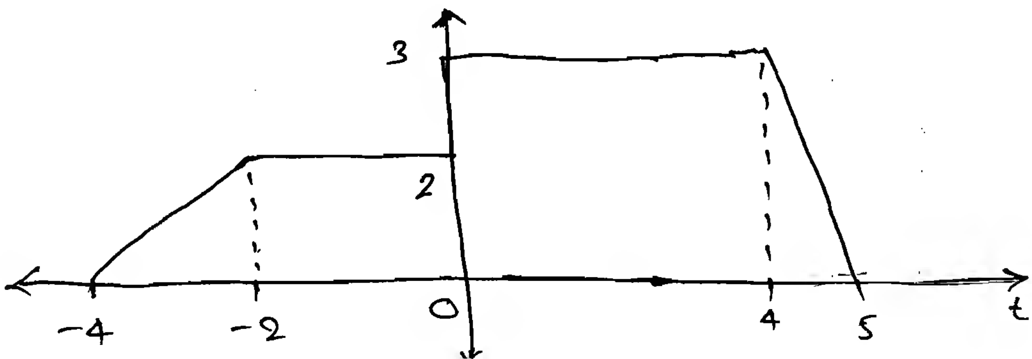
$-2 < t < -1 \Rightarrow$ Slope = $0+1=1$.

$-1 < t < +1 \Rightarrow$ Slope = $1-1=0$.

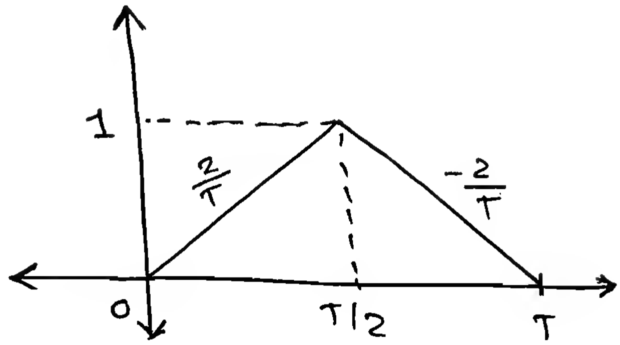
$1 < t < 2 \Rightarrow$ Slope = $0-1=-1$.

$2 < t < \infty \Rightarrow$ Slope = $-1+1=0$.

* Write the following signal in terms of singularity f^{n}s:



Solⁿ: $y(t) = 1 \cdot r(t+4) - 1 \cdot r(t+2) + u(t) - 3r(t-4) + 3r(t-5).$

② 

Solⁿ: $y(t) = \frac{2}{T} r(t) - \frac{4}{T} r\left(t - \frac{T}{2}\right) + \frac{2}{T} r(t-T).$

③ If $x(t) = 0$ for $t < 3$. For what range of t following fⁿ are said to zero.

① $x(1-t) + x(2-t) = 0$ for t?

② $x(1-t) \cdot x(2-t) = 0$ for t?

Solⁿ: $x(t) = 0$ for $t < 3$.

→ $x(1-t) = 0$; $1 - t < 3$

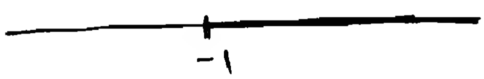
$-t < 2$

$x(1-t) = 0$; $t > -2$

→ $x(2-t) = 0$; $2 - t < 3$

$-1 < t$

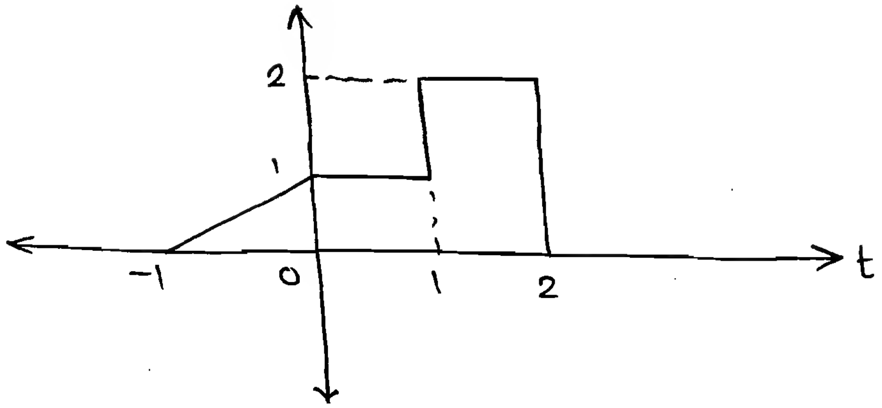
$x(2-t) = 0$; $t > -1.$



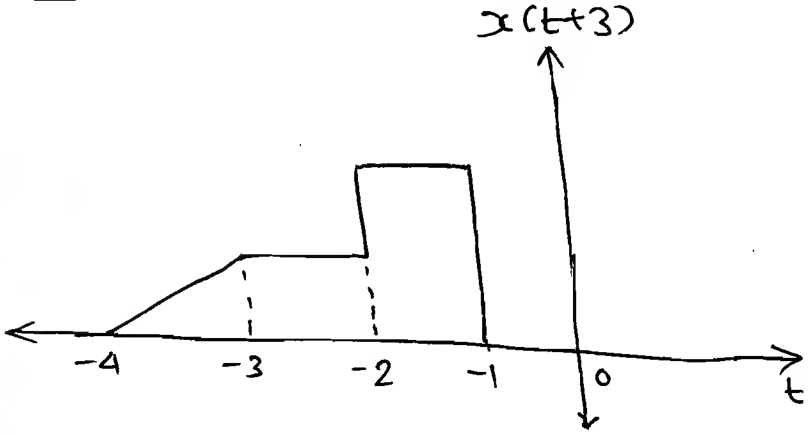
① $x(1-t) + x(2-t) = 0$ for $t > -1$.

② $x(1-t) \cdot x(2-t) = 0$ for $t > -2$.

* For the signal $x(t)$ shown in figure. Draw the following signals:



Sol^n: (1) $x(t+3)$



* $x(at+b)$.

⇒ Method: -(1): $x(t) = x\left(a\,(t \pm b/a)\right)$.

(i) Time - scaling $x(at)$.

(ii) Shift - $x(at)$ by $\pm b/a$.

⇒ Method: -(2):

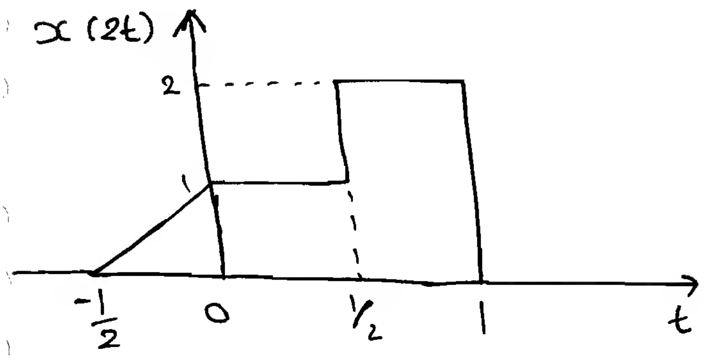
$$x(t) \longrightarrow x(t \pm b) \longrightarrow x(at \pm b).$$

(i) Time shift by b i.e $x(t \pm b)$.

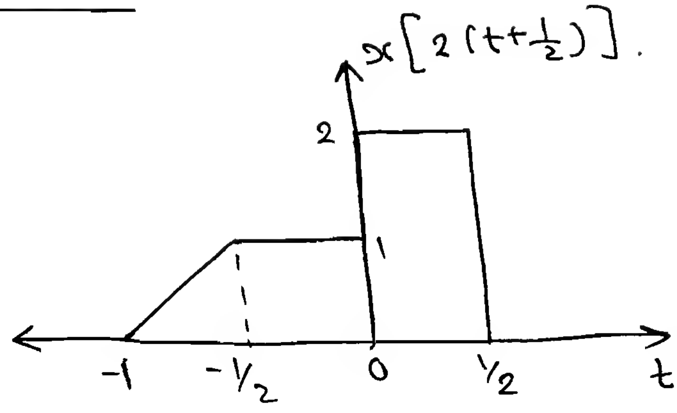
(ii) time - scaling $x(t+b)$ by $x(at \pm b)$.

② $x(2t+1)$.

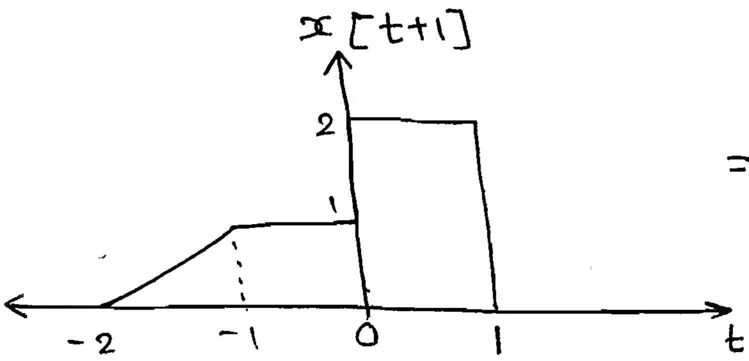
Sol^n: Method - 1 : $x[2(t+\frac{1}{2})]$.



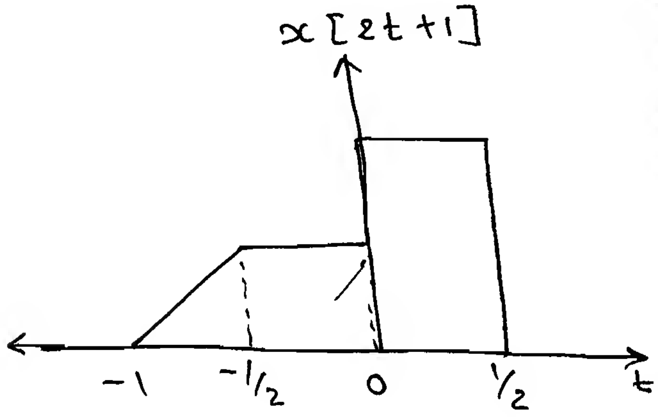
$\Rightarrow$



$\Rightarrow$ Method - 2 : $x[2t+1]$

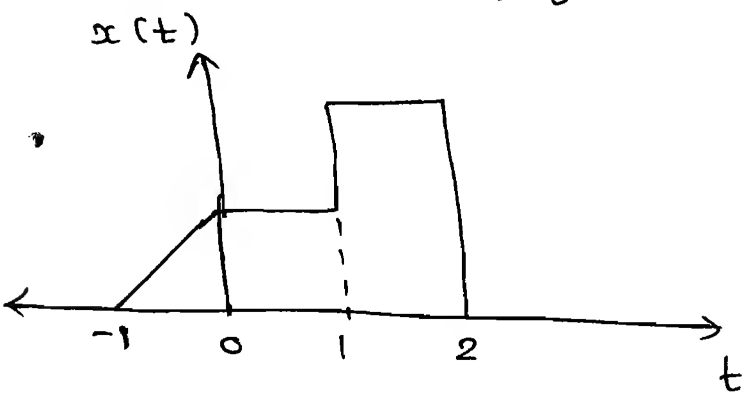


$\Rightarrow$

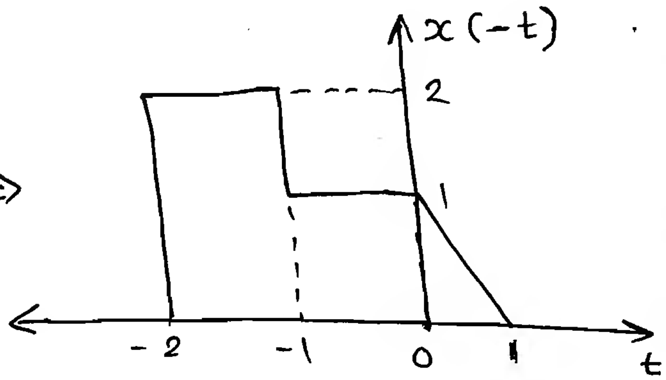


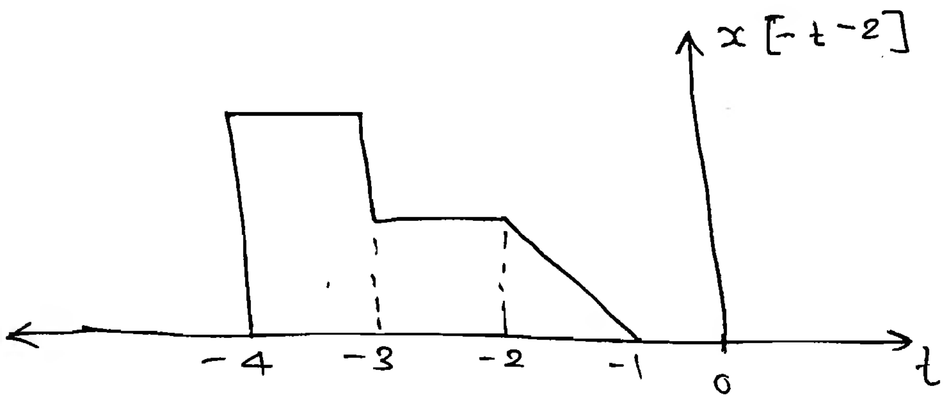
③ $x(-t-2)$.

$x[-(t+2)]$



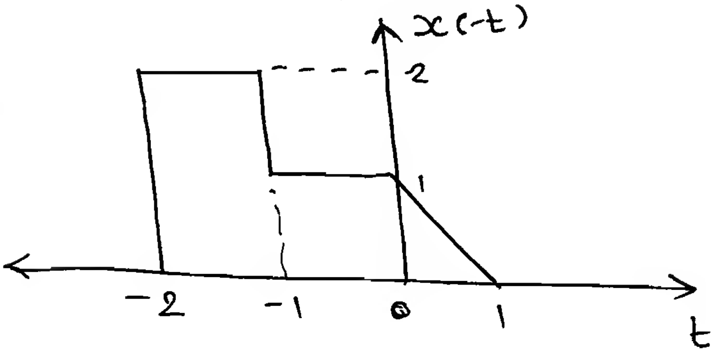
$\Rightarrow$

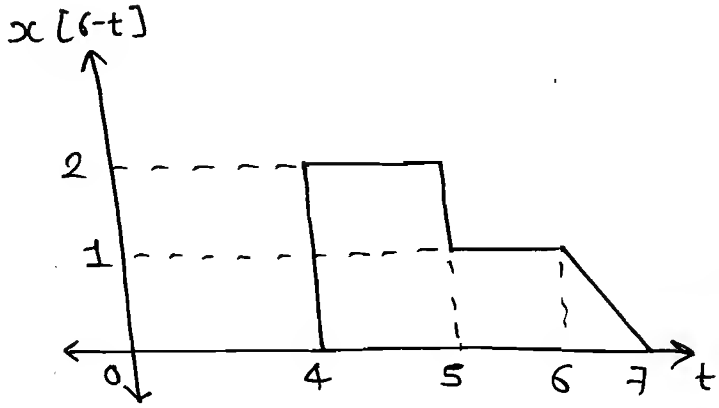




④ $x[6-t]$.

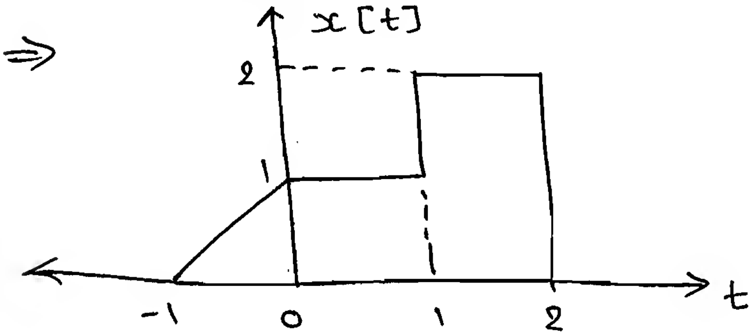
Sol^n: $x[-[t-6]]$.

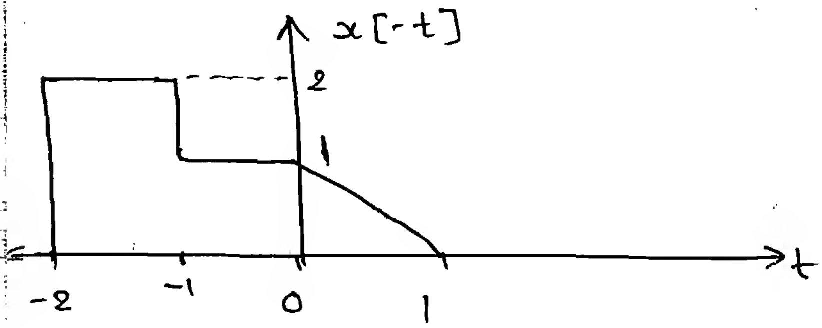


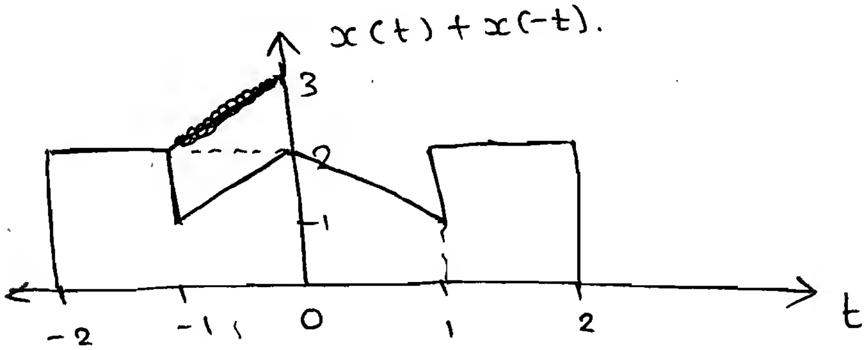


⑤ $[x(t) + x(-t)]u(t)$.

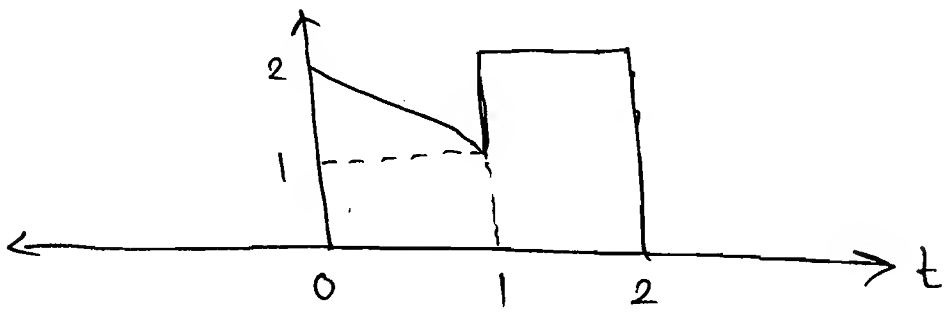
⇒







$\times u(t)$



$[x(t)+x(-t)]u(t)$.

$*\quad x(1 - t/3) = x\left[-\frac{1}{3}(t-3)\right].$

$$\Rightarrow x(t) \xrightarrow{T.R.} x(-t) \xrightarrow{T.S.} x(-t/3) \xrightarrow{R.S.} x\left((t-3)\cdot -\tfrac{1}{3}\right).$$

Can be Interchange

Can not be Interchange

$\Rightarrow$

$$y(t) = x(t-t_0)$$

$$y(\alpha t) = x(\alpha t - t_0) \checkmark$$

$$= x(\cancel{\alpha(t-t_0)})$$

$$y(t) = x(\alpha t).$$

$$y(t-t_0) = x(\alpha(t-t_0)) \checkmark$$

$$= x(\cancel{\alpha t} - t_0).$$

$\boxed{Q}$ Given, $x[n] = (6-n)[u(n) - u(n-6)]$.

Draw the following signals

(i) $x[n+3]$

(ii) $x[6-n]$

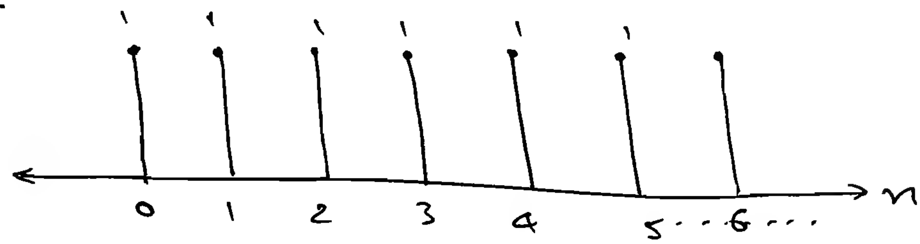
(iii) $x[3n+1]$

(iv) $x\left[\frac{n}{2}\right]$

(v) $x[n-1]\cdot\delta[n-3]$

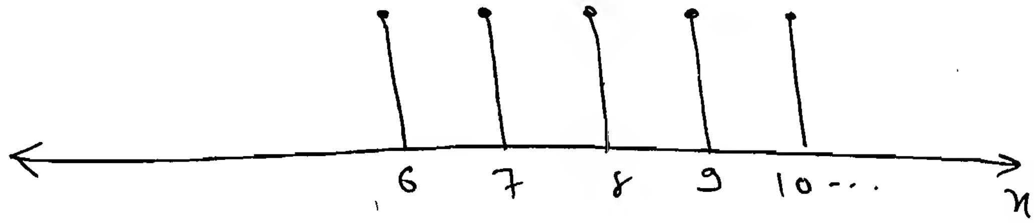
Soln:

$u[n]$

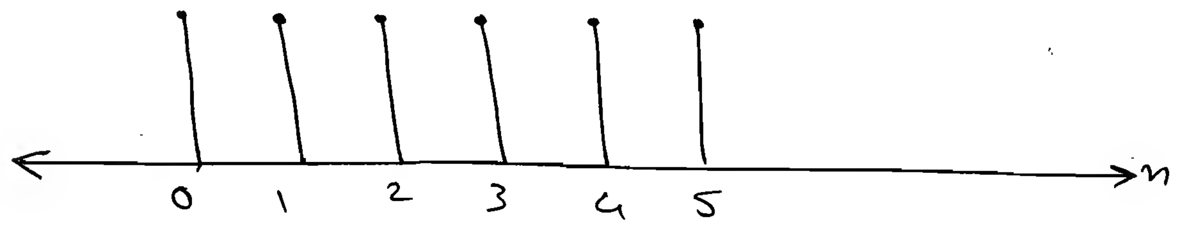


$-$

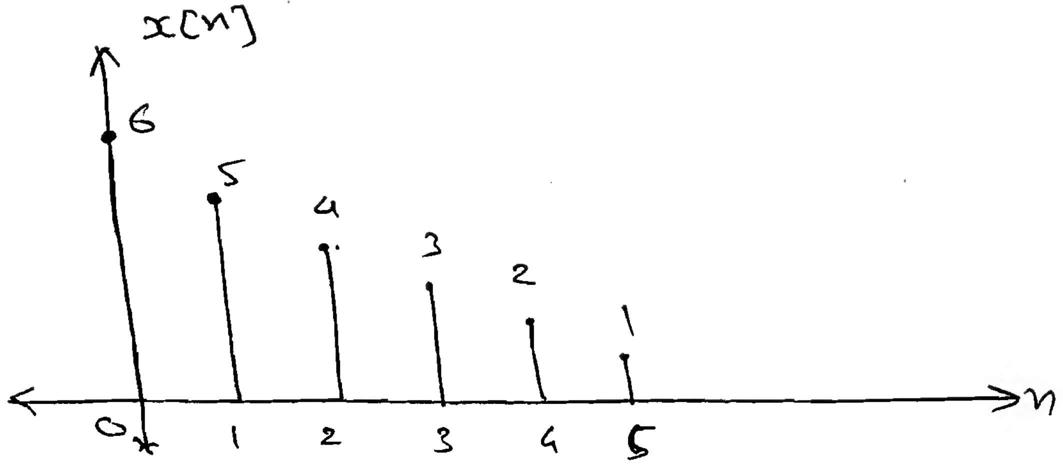
$u[n-6]$



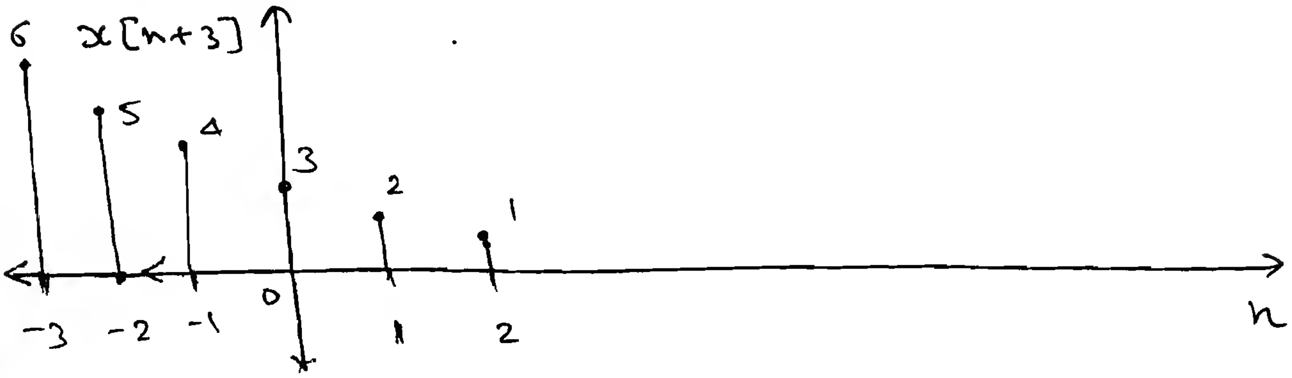
$u[n] - u[n-6]$.



⇒



① $x[n+3]$:



② $x[6-n]$.

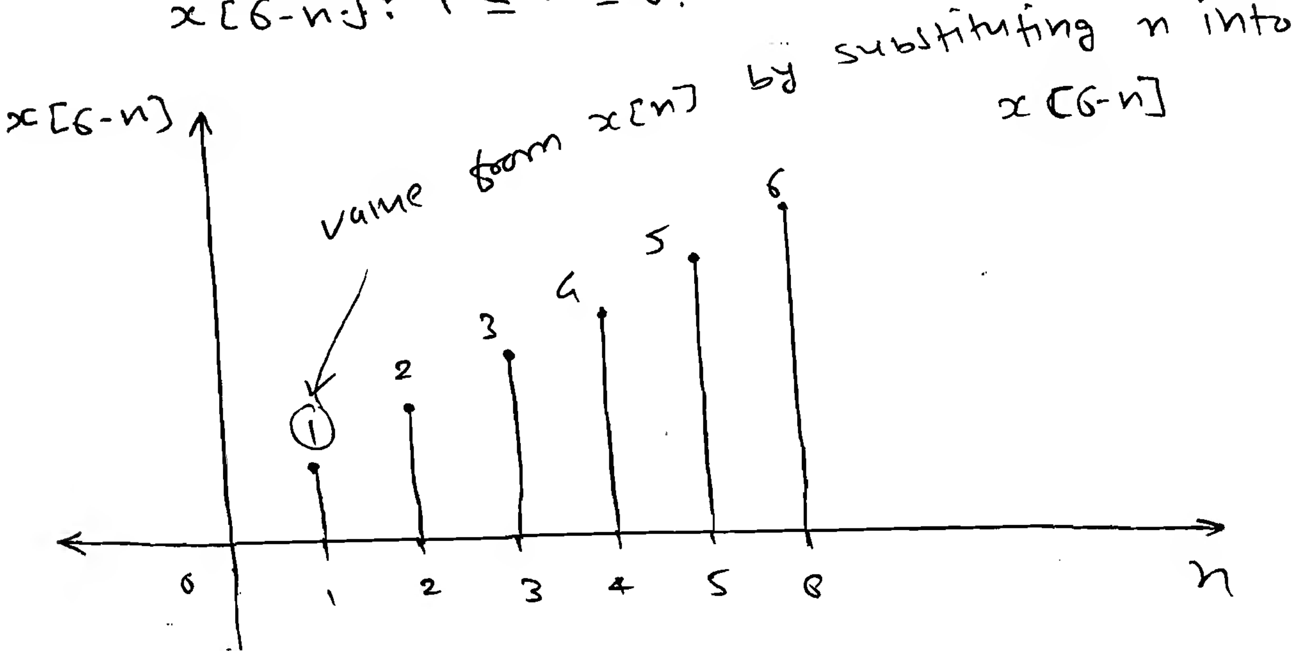
$$\Rightarrow \quad x[n]: \quad 0 \le n \le 5$$

$$x[6-n]: \quad 0 \le 6-n \le 5$$

$$-6 \le -n \le -1$$

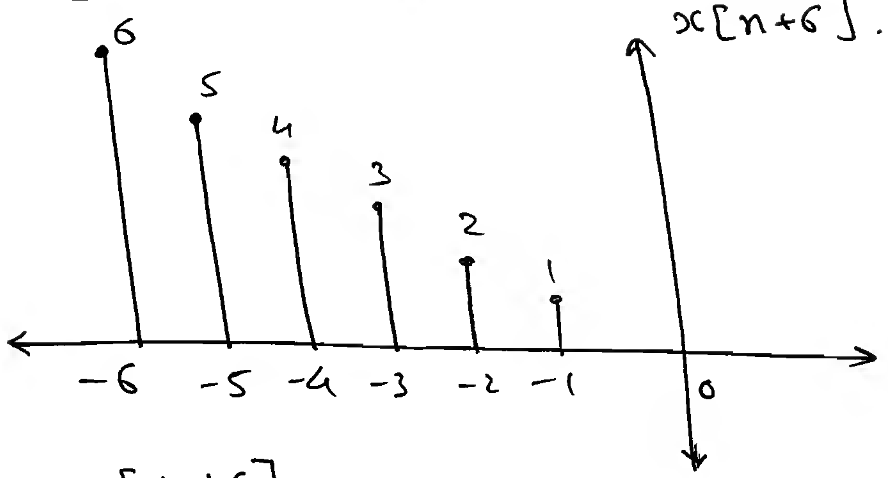
$$6 \ge n \ge 1$$

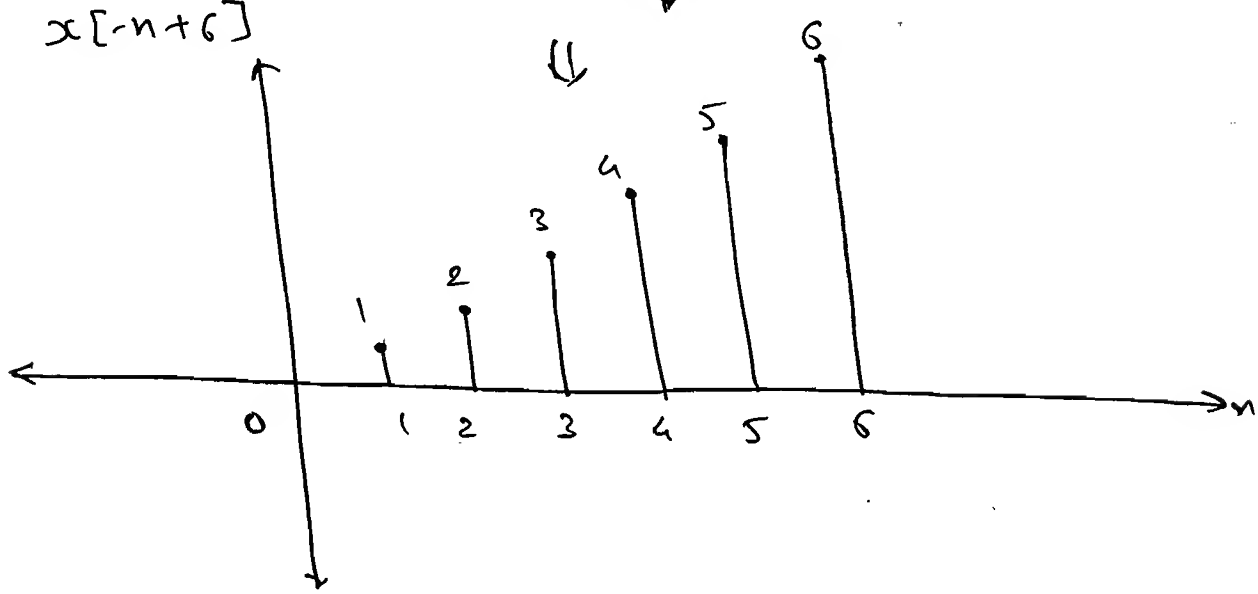
$$x[6-n]: \ 1 \le n \le 6.$$



Method-2:

1. shift left by 6
2. Time reversel.





3. $x[3n+1]$.

$\Rightarrow$ $\cancel{x\left[3\left(n+\frac{1}{3}\right)\right]}$ $\cancel{n_0 = -1/3}$.

NOTE: Method - I i.e. first scaling and then shifting may be fails in discrete time domain. because n is integer. so, always go through by following two methods.

Method -I: $x[n]$: $0 \le n \le 5$.

$\therefore x[3n+1]$: $0 \le 3n+1 \le 5$

$-1 \le 3n \le 4$

$\therefore -\frac{1}{3} \le n \le 4/3$.

Valid index,

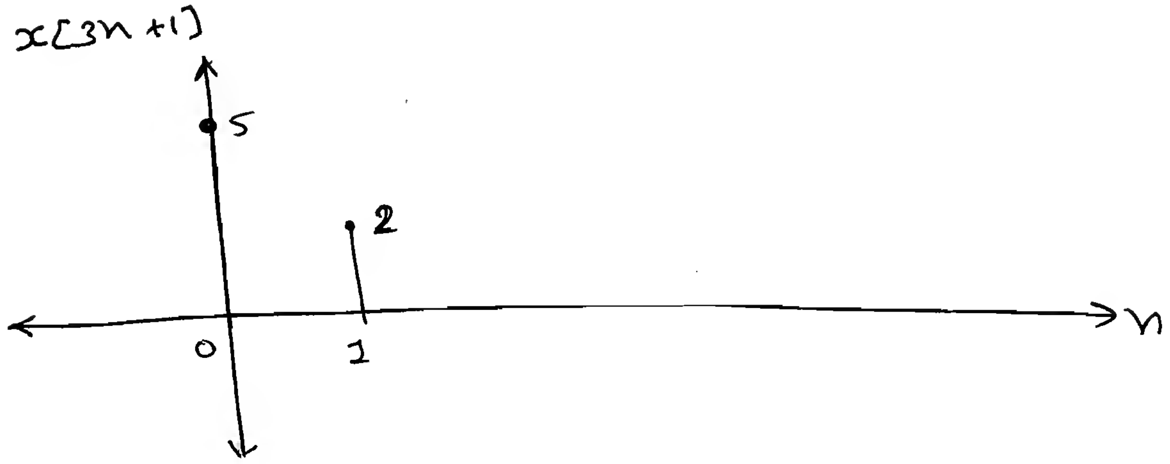
$-0.33 \leq n \leq 1.33.$

but n is always integer.

So, valid n in range $(-0.33, 1.33)$

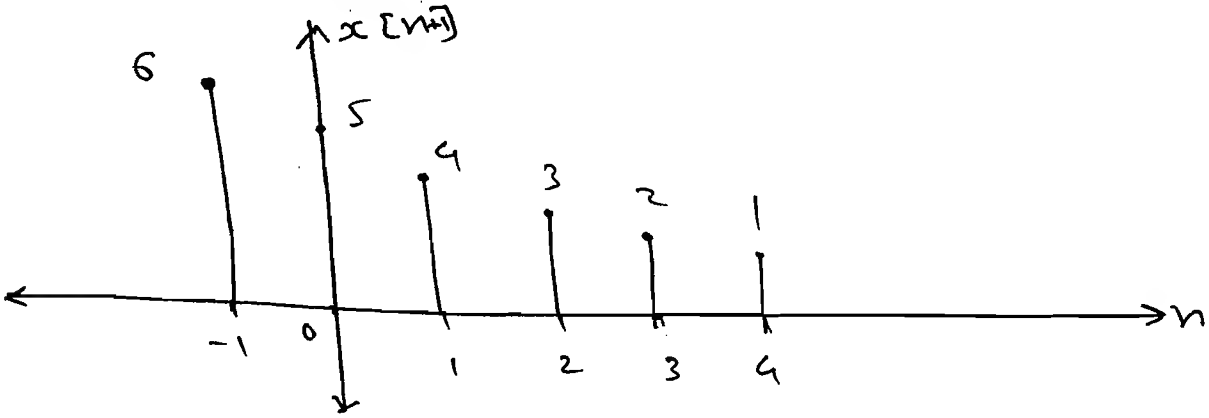
is $n=0$ & $n=1$.

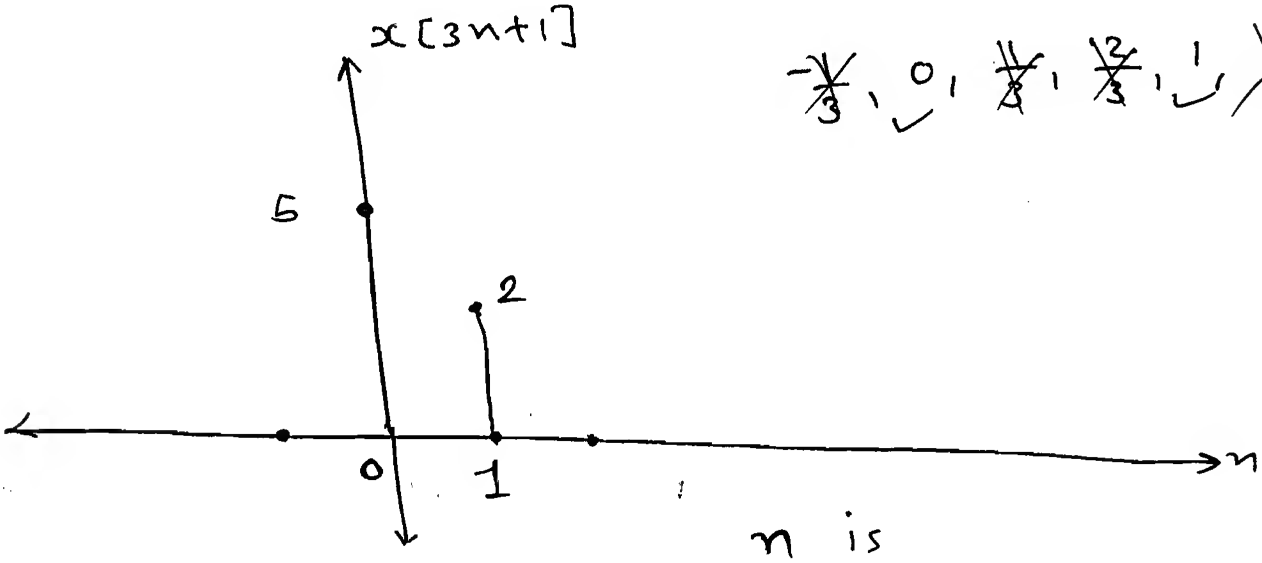
$\therefore$



<u>Method - II</u>:

$\rightarrow$ (1) shifting (2) scaling.





$\cancel{-\frac{1}{3}}, 0, \cancel{\frac{1}{3}}, \cancel{\frac{2}{3}}, 1, \cancel{\frac{4}{3}}$

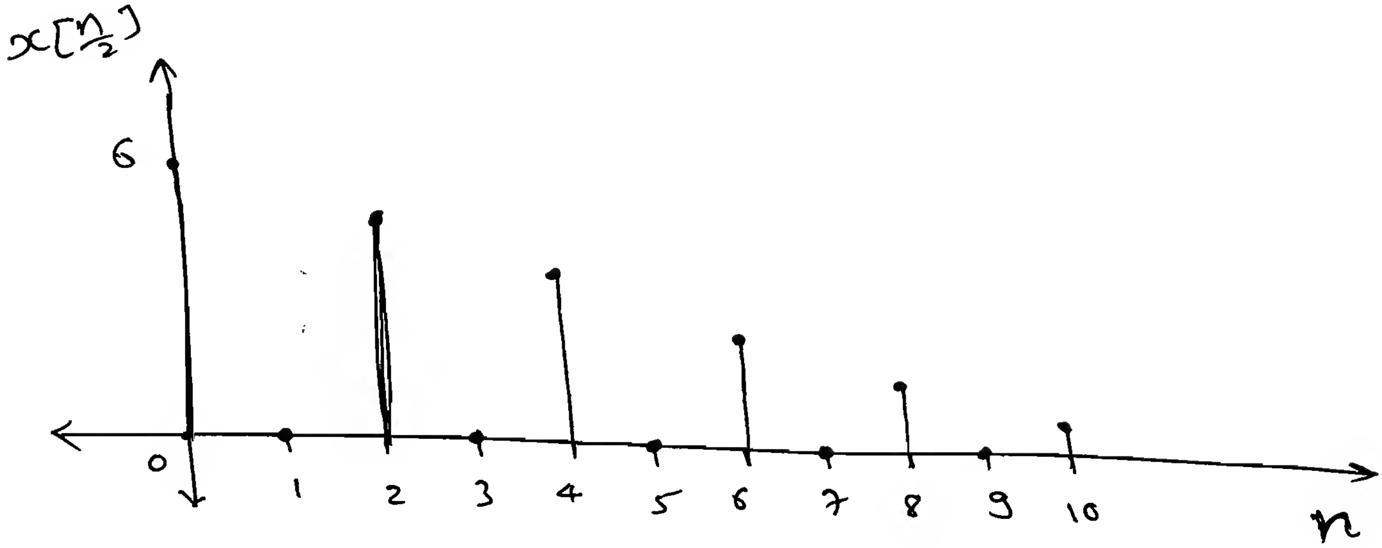
n is

④ $x[\frac{n}{2}]$.

$\Rightarrow$ $x[n]$: $0 \le n \le 5$

$x[\frac{n}{2}]$: $0 \le \frac{n}{2} \le 5$

$0 \le n \le 10$.



* Interpolation and Decimation.

$\Rightarrow$ In cont^n time domain,

$x[at] \rightarrow$ Scaling property,

a is scaling factor.

if $a > 1 \Rightarrow$ Compression

$a < 1 \Rightarrow$ Expansion.

$\Rightarrow$ In ~~con~~ discrete time domain,

$x[mn] \rightarrow$ scaling property.

if $m > 1 \Rightarrow$ Decimation.

$m < 1 \Rightarrow$ Interpollation.

$\Rightarrow$ In above ~~expa~~ example, we can say

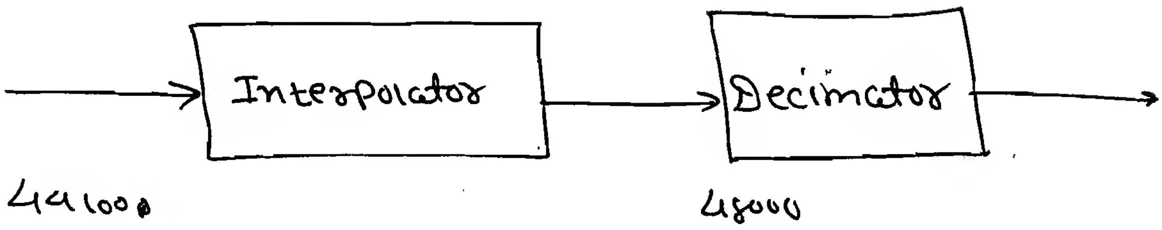
Zero - interpolation because it makes some

Sample zero.

⇒ **Multi- Rate DSP** :-

⇒ C.D. : $f_s = 44.1$ KHz.

D.A.T. : $f_s = 48$ KHz.

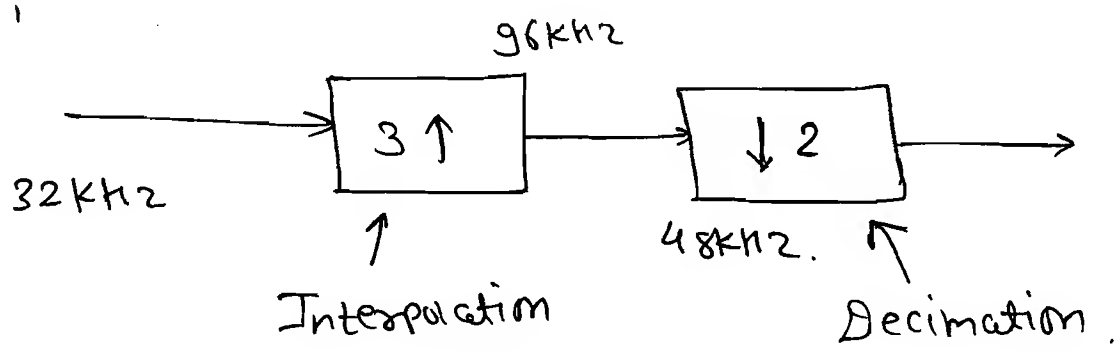


→ Let, f_{s_1} : 32 KHz

f_{s_2} : 48 KHz.

Now, we want to convert 32K S.R. to the 48 K S.R.

So,



$$\frac{48}{32} = 3/2 \Rightarrow x\left[\frac{2}{3}n\right].$$

NOTE:

⇒ Conversion of one samping rate to another samping rate is multiRate DSP which is done by using decimator and

interpolator.

→ Whenever we want to perform this operations simultaneously first do Interpolation and then Decimation.
(Up sampling) (Down sampling).

i.e. if $x\left[\frac{an}{b}\right]$ → then first do $x\left[\frac{n}{b}\right]$ and then $x\left[\frac{an}{b}\right]$.

→ In interpolation no. of samples increases and in Decimation no. of samples Decreases.

(5) $x[n-1]\,\delta[n-3]$.

$\Rightarrow$ $x[n-1]\,\delta[n-3]$

$n_0 = 3$.

∴ by sifting property of δ

$x[3-1]\cdot\delta[n-3]$.

$= \cancel{4}1\cdot\delta[n-3]$.

$\boxed{Q}$ If $x[n] = 1 - \sum_{k=4}^{\infty} \delta[n-1-k]$. Such that $x[n] = u[Mn - n_0]$ find M & n_0.

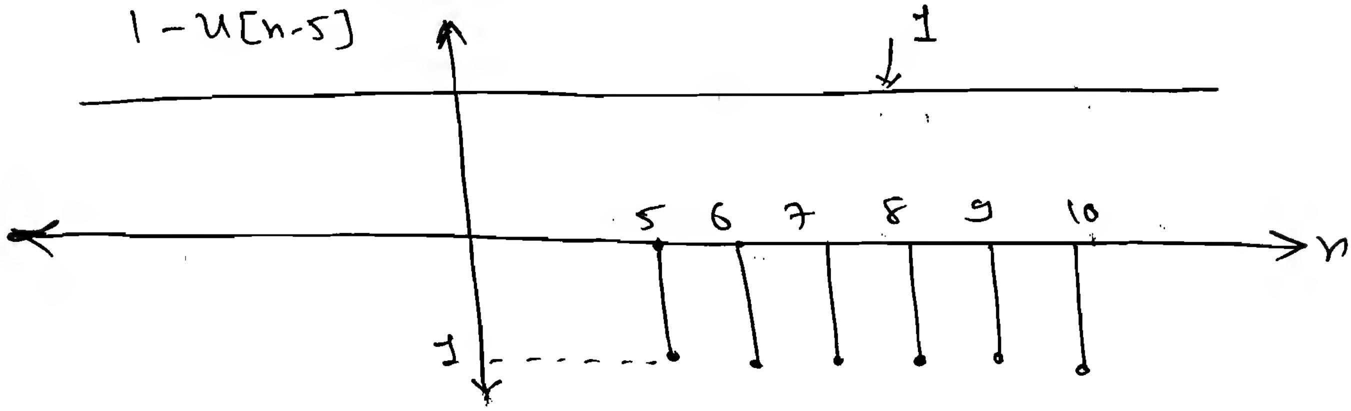
Sol^n:

$x[n] = 1 - [\delta[n-5] + \delta[n-6] + \delta[n-7] + \dots]$.

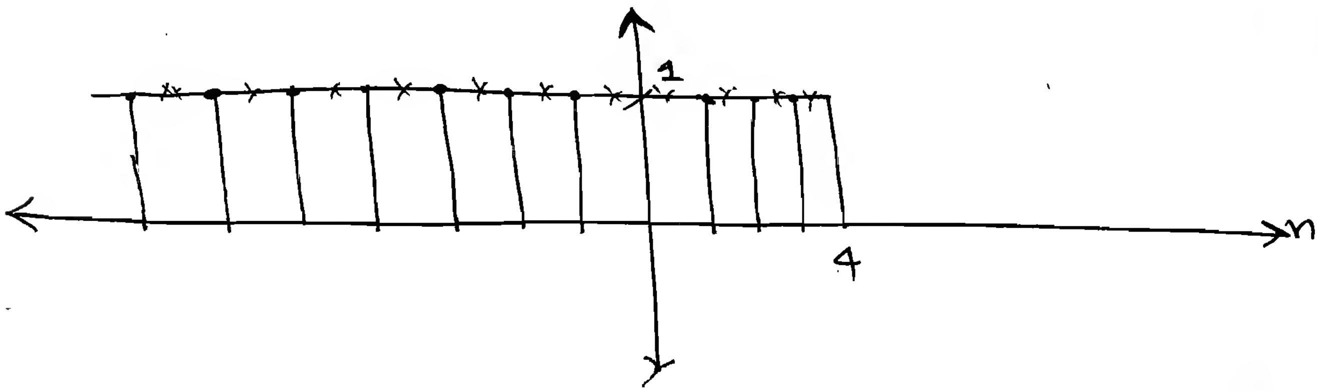
$x[n = 1 - u[n-5]$

$\Rightarrow$

$\Rightarrow$



$\Downarrow$



So, $u[-n+4]$.

So, $m = -1$, $n_0 = -4$.

☆ Classification of signals: $\begin{cases} x(t) \\ x[n] \end{cases}$

① Energy and power signal:

⇒ **Energy signal:**

→ If Energy of a signal is finite then it is called Energy ~~sing~~ signal i.e $0 < E < \infty$.

⇒ For cont^n time signal,

$$\boxed{E_{x(t)} = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \cdot dt.} = \text{finite.}$$

⇒ For discrete time signal,

$$\boxed{E_{x[n]} = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2} = \text{finite.}$$

⇒ For Complex $|x(t)|^2 = x(t) \cdot x^*(t)$.

⇒ **Power signal:**

⇒ If avg. power of a signal is finite i.e. $0 < P < \infty$ then it is called power signal.

$$\boxed{P_{av\,x(t)} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \cdot dt.}$$

For discrete time signal,

$$\boxed{P_{avg\,x[n]} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x(n)|^2}$$

n: $-\infty$ to $+\infty$

: $-N$ to $+N$

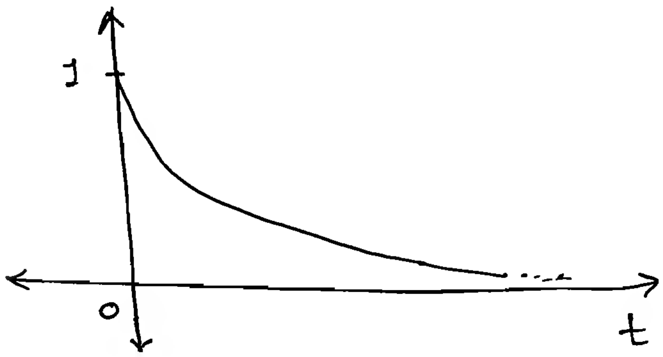
$-N$ to -1, 0, 1 to N

So, $2N+1$.

⇒ The physically possible (or) Practically possible signal is said to be the Energy signal if following condn is satisfied:

$$\boxed{\left.\begin{array}{l} \text{As } t \to \pm\infty \Rightarrow \underline{t \to \pm\infty}. \\ \underline{\text{amp} \to 0} \end{array}\right\} \text{Energy signal}}$$

* $\underline{x(t) = e^{-t}\cdot u(t)}$.



$$E_{x(t)} = \lim_{T\to\infty} \int_0^T e^{-2t}\,.dt$$

$$= \lim_{T\to\infty} \left[\frac{e^{-2t}}{-2}\right]_0^T$$

$$= \lim_{T\to\infty} \frac{1-e^{-2T}}{2}$$

$$= \frac{1}{2} . = \text{finite},$$

So, Energy signal.

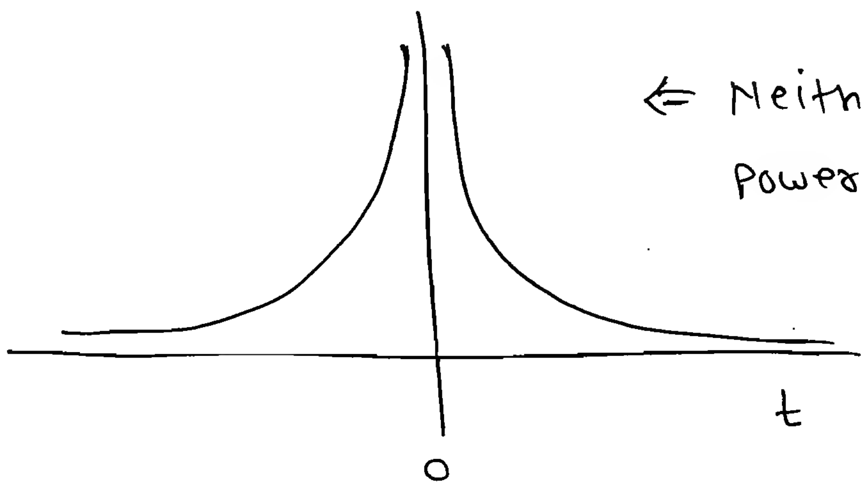
$$\Rightarrow P_{av} = \lim_{T\to\infty} \frac{E}{2T} = \frac{1}{\infty} = 0.$$

So, An Energy signal should has zero avg. Power.

→ over the signal is existing it should maintain the finite amplitude level.

* $\underline{y(t) = \left|\frac{1}{t}\right|}$.

⇒



⇐ Neither Energy nor Power signal.

⇒ at $t=0$ its amplitude is ∞.

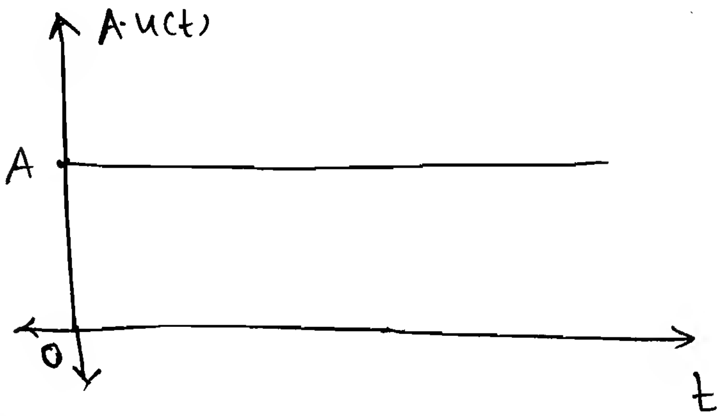
⇒ at $t=\infty$ it's amplitude is ZERO.

So, this type of signal can't be generated practically.

⇒ **Power signal:**

⇒ A signal which maintain constant amplitude over infinite time is a power signal.

e.g. $y(t) = A \cdot u(t)$.



$$P_{av} = \lim_{T\to\infty} \frac{1}{2T} \int_0^T A^2 \cdot dt$$

$$P_{av} = \lim_{T\to\infty} \frac{1}{2\cancel{T}} \times A^2 \cdot \cancel{T}.$$

$$= \lim_{T\to\infty} \frac{A^2}{2}.$$

$$\therefore \boxed{P_{av} = \frac{A^2}{2}.}$$

Now, $P_{av} = \lim_{T\to\infty} \frac{1}{2T} \cdot E.$

$$\Rightarrow E = \lim_{T\to\infty} 2T \cdot (P_{av}).$$

$$E = \lim_{T\to\infty} 2T \cdot A^2/2$$

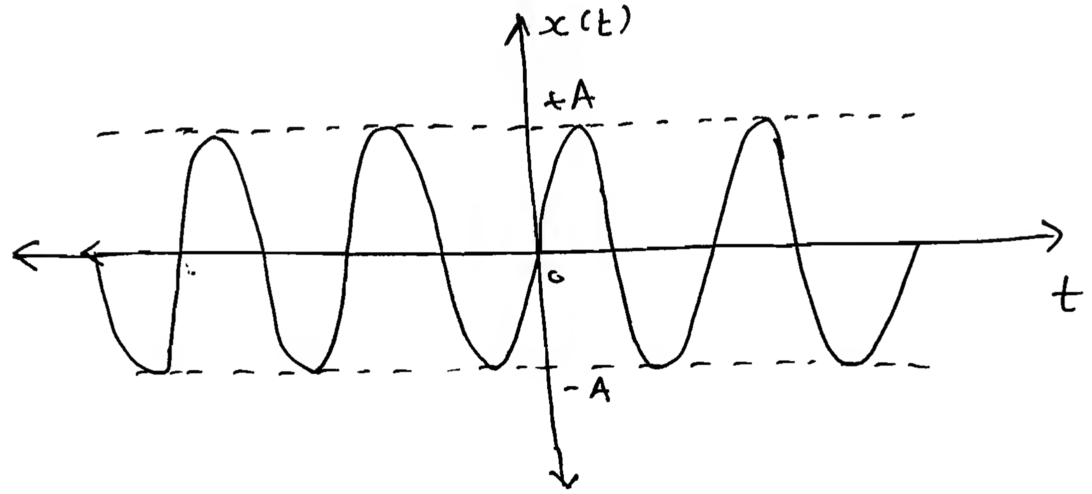
$$\boxed{E = \infty}$$

⇒ So, Power signal require ∞ energy.

⇒ The signal cannot be both energy and power signal. Mutually Exclusive.

$\Rightarrow$ All Periodic signals are Power signals because they maintain constant amplitude over a infinite time.

for. e.g. (i) $x(t) = A\cos(\omega_0 t + \theta)$.



$$P_{av} = \left(\frac{A}{\sqrt{2}}\right)^2 = (rms)^2 = (\text{mean square})^2.$$

$$\therefore\ P_{av} = \frac{A^2}{2}.$$

(ii) $x(t) = A \cdot e^{j\omega_0 t}$

$$P_{av} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |A \cdot e^{j\omega_0 t}|^2 \cdot dt.$$

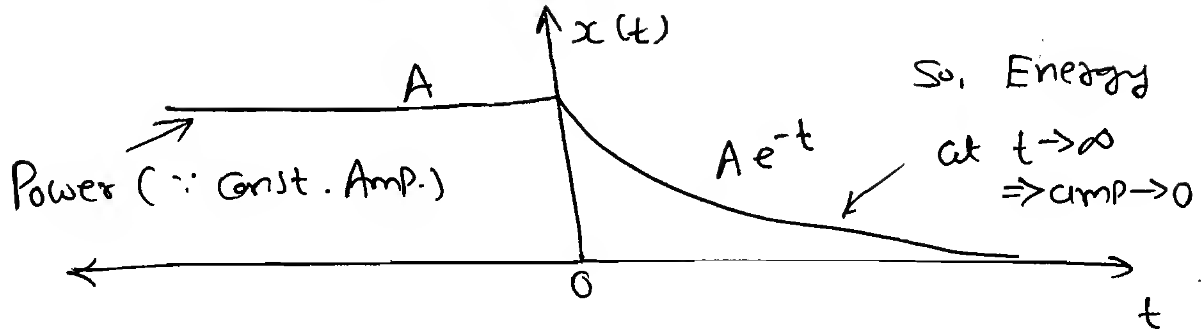
$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} A^2 \cdot |e^{j\omega_0 t}|^2 \cdot dt.$$

$$= \lim_{T\to\infty} \frac{1}{\cancel{2T}} \cdot A^2 \cdot \cancel{2T}$$

$$P_{av} = A^2.$$

$\Rightarrow$ Complex sinusoidal is also a power signal.

⇒ What is Nature of signal below:



$$\Rightarrow P_{av} = \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt.$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\int_{-T}^{0} A^2 \cdot dt + \int_{0}^{T} A^2 \cdot \cancel{|e^{-2t}|^2} \cdot dt \right]$$

0

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{0} A^2 \cdot dt$$

$$P_{av.} = \frac{A^2}{2}$$

case-(i): if power signal then E signal has zero power so power + 0 = power

case-(ii) if E signal then P has infinite E.
so, E = ∞ means at ~~t → ∞ ⇒ E → ∞~~

So, Power + Energy = Power.

$\boxed{Q}$ If $x[t] = \delta(t+1) - \delta(t-3)$. Find the energy in $y[t] = \int_{-\infty}^{t} x(\tau)\, d\tau$.

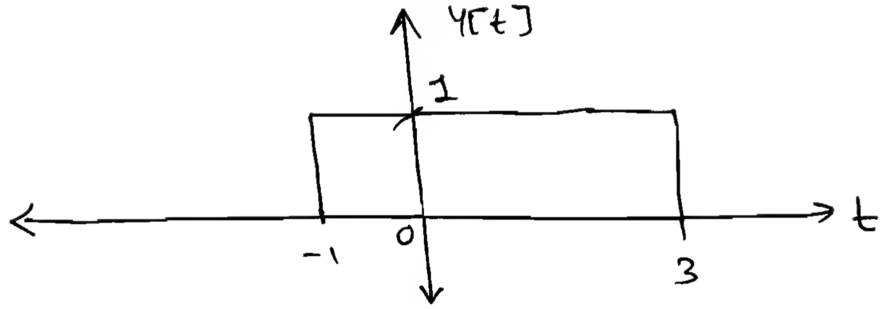
Solⁿ:

$$y(t) = \int_{-\infty}^{t} (\delta(t+1) - \delta(t-3))\, d\tau.$$

$$= u[t+1] - u[t-3].$$

$$= u[t+1) - u(t-3).$$

⇒



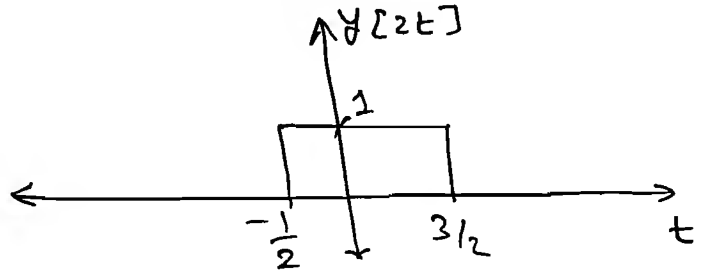
⇒ $E_{y(t)} = \int_{-1}^{3} (1)^2 \cdot dt = 4$

$\therefore \boxed{E_{y(t)} = 4.}$ — ⊛

⇒ All finite duration signal of finite amplitude are energy signals.

⇒ for the above example find E for $y[2t]$.

⇒



⇒ $E_{y(2t)} = \int_{-1/2}^{3/2} (1)^2 \cdot dt = \frac{4}{2} = 2.$

$\therefore \boxed{E_{y(2t)} = 2}$ — ⊛

⇒ $\boxed{E\ y(\alpha t) = \frac{E_{y(t)}}{|\alpha|}}$. Imp. ✱

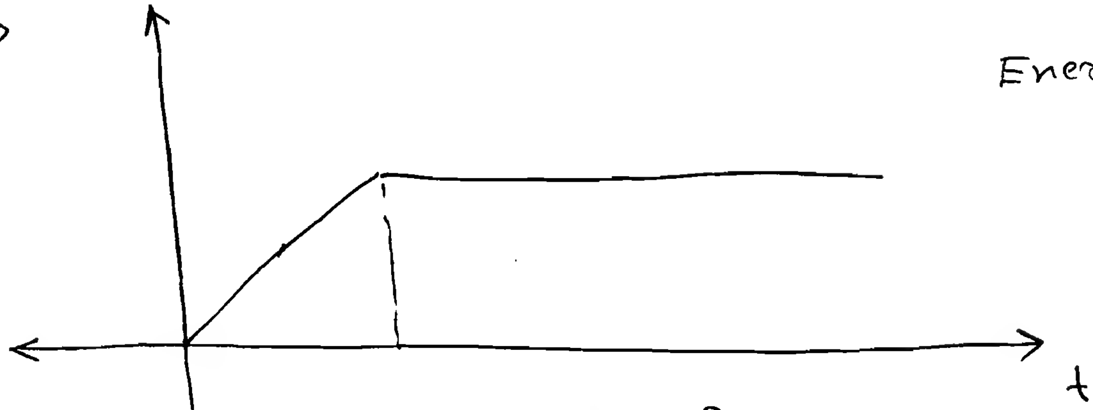
⇒ Imp.

	Energy
$A\,y(t) \Longrightarrow$	$A^2 E_{y(t)}$.
$A + y(t) \Longrightarrow$	$\infty = E$
Power + Energy	= Power

* $x(t) = r(t) - r(t-1)$.

For power → see constant amp.
Energy → verify $t \to \pm\infty \Rightarrow Amp = 0$.

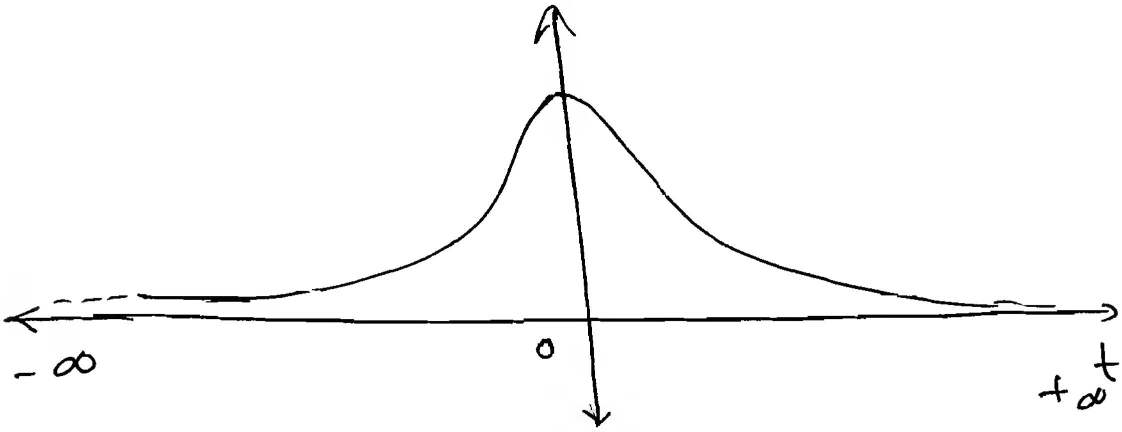
⇒



Energy + Power = Power.

* $x(t) = e^{-\pi t^2}$

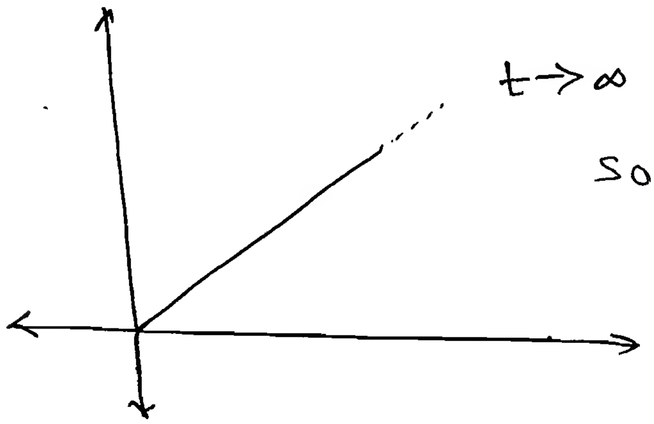
⇒



Energy signal

* $x(t) = t\,u(t)$.

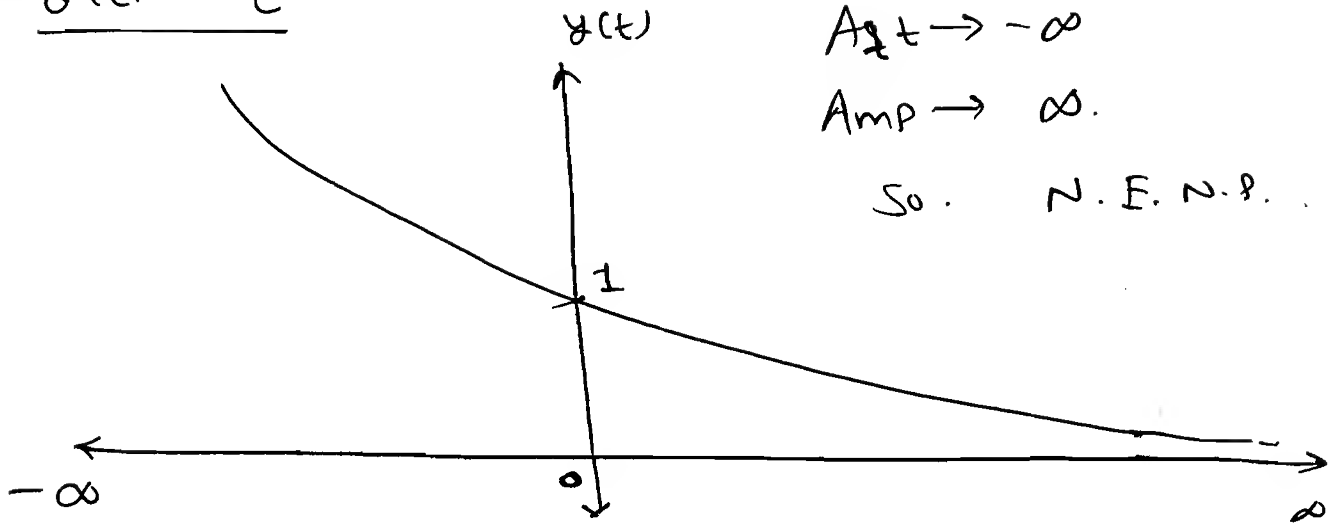
⇒



$t \to \infty \Rightarrow Amp \to \infty$

So, NENP.

* $y(t) = e^{-3t}$.



As $t \to -\infty$

Amp $\to \infty$.

So. N.E.N.P.

> To make Energy signal,
> amp at $t \to -\infty$ & $t \to +\infty$ must be zero.

Note:

⇒ Contn impulse function is N.E.N.P. signal,

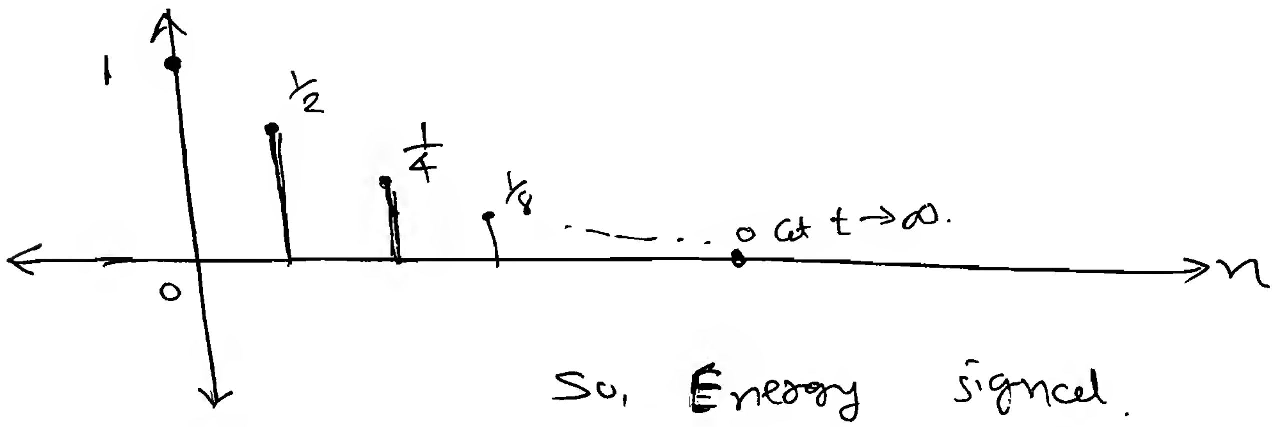
⇒ Discrete impulse is Energy signal.

* **Discrete Signal:**

⇒ $x[n] = \alpha^n u[n]$.

(i) For $|\alpha| < 1$. $\longrightarrow$ Energy.

$x[n] = \left(\frac{1}{2}\right)^n u[n]$.



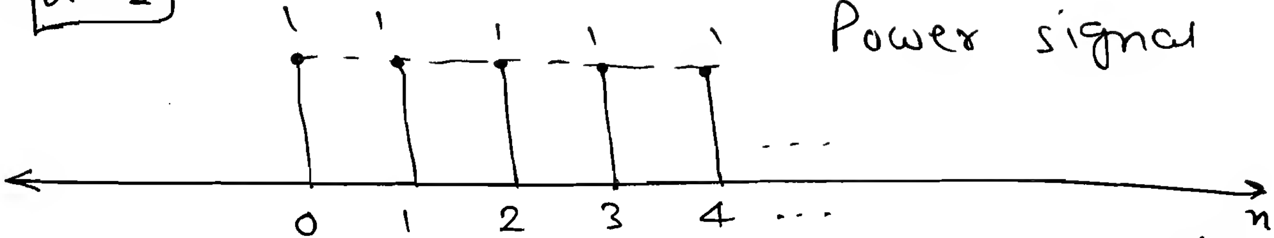
So, Energy signal

as $t \to \infty \Rightarrow$ amp $= 0$.

(ii) $|\alpha| = 1 \longrightarrow$ Power

$x[n] = u[n]$

* $\boxed{\alpha = 1}$



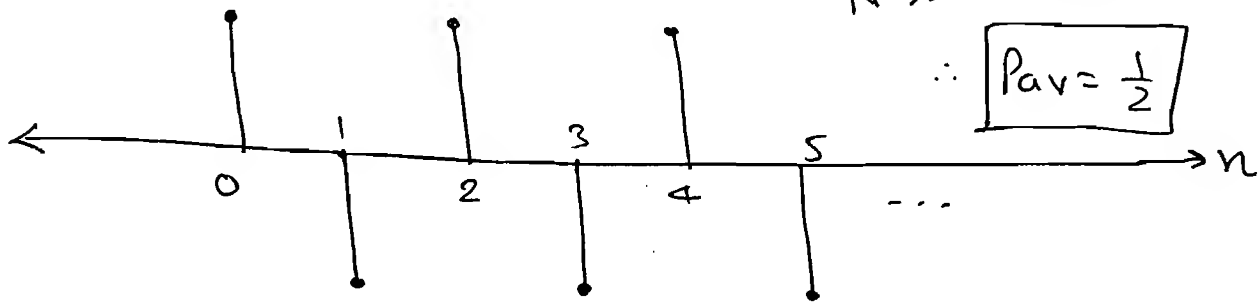
Power signal

$$P_{av} = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)^2$$

$$= \lim_{N \to \infty} \frac{N+1}{2N+1} = \frac{1}{2}$$

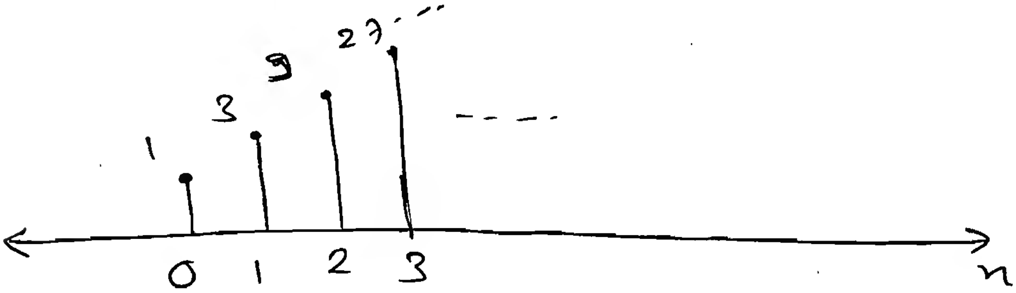
$\therefore \boxed{P_{av} = \frac{1}{2}}$

* $\boxed{\alpha = -1}$



(iii) $|\alpha| > 1 \longrightarrow$ NENP.

$x[n] = 3^n . u[n]$



As $n \to \infty$

amp $\to \infty$.

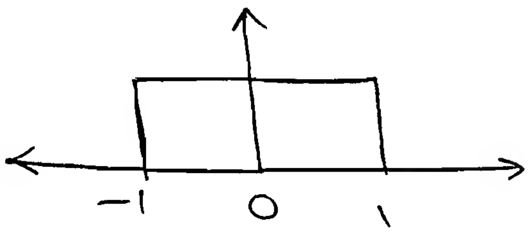
So, N.E.N.P.

[2] Even & Odd signals.

① Even signals:

$\Rightarrow$ $\boxed{x(t) = x(-t).}$

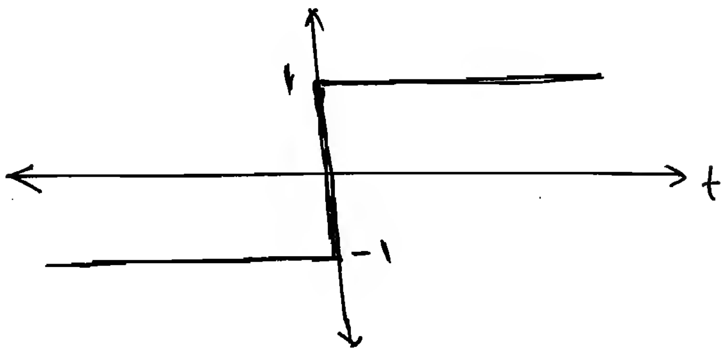
for e.g.



② Odd signals:

$\Rightarrow$ $\boxed{x(t) = -x(-t).}$

for. eg. signum fn.



$\Rightarrow$ For Conjugate signals,

$$x(t) = a(t) + j\,b(t).$$

Even Conjugate : $\boxed{x(t) = x^*(-t).}$

e.g.: $x(t) = t^2 \cdot e^{j\omega t}$

$x(-t) = t^2 \cdot e^{-j\omega t}$

$x^*(-t) = t^2 \cdot e^{j\omega t} = x(t).$

→ **Odd Conjugate:**

$$\boxed{x(t) = -x^*(-t).}$$

for e.g.: $x(t) = t \cdot e^{j\omega t}$.

$\Rightarrow \quad x(t) = x_e(t) + x_o(t)$. — (I).

Replace 't' by '-t'

$\therefore \quad x(-t) = x_e(-t) + x_o(-t)$.

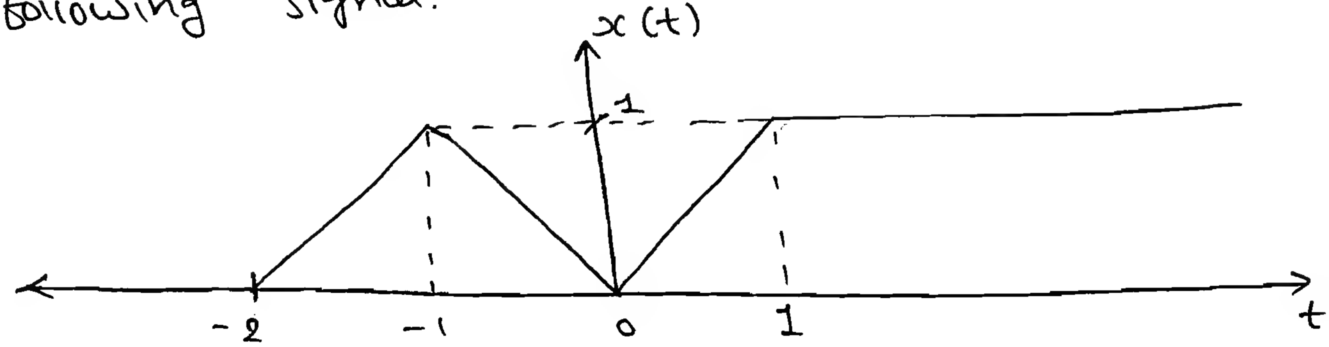
$\Rightarrow \quad x(-t) = x_e(t) - x_o(-t)$ — (II).

$\Rightarrow$ from eqn (I) & (II).

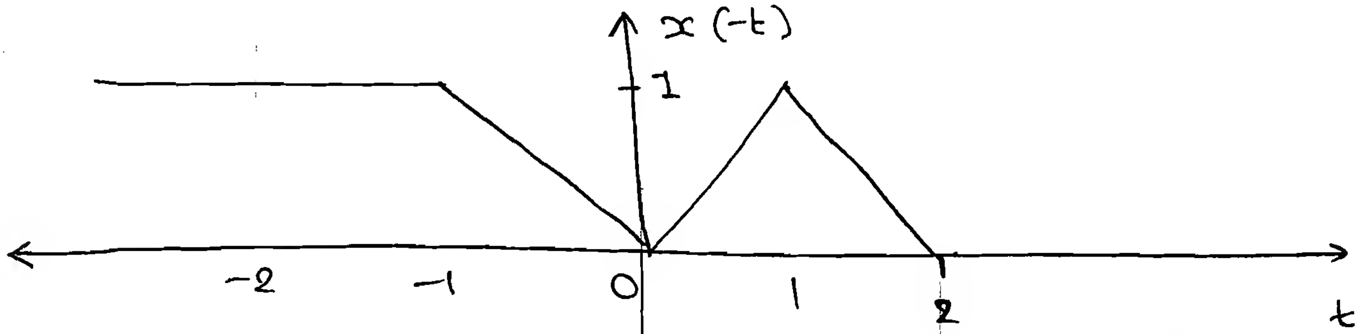
$$\Rightarrow \boxed{\begin{aligned} x_e(t) &= \frac{x(t) + x(-t)}{2} \\ x_o(t) &= \frac{x(t) - x(-t)}{2} \end{aligned}}$$

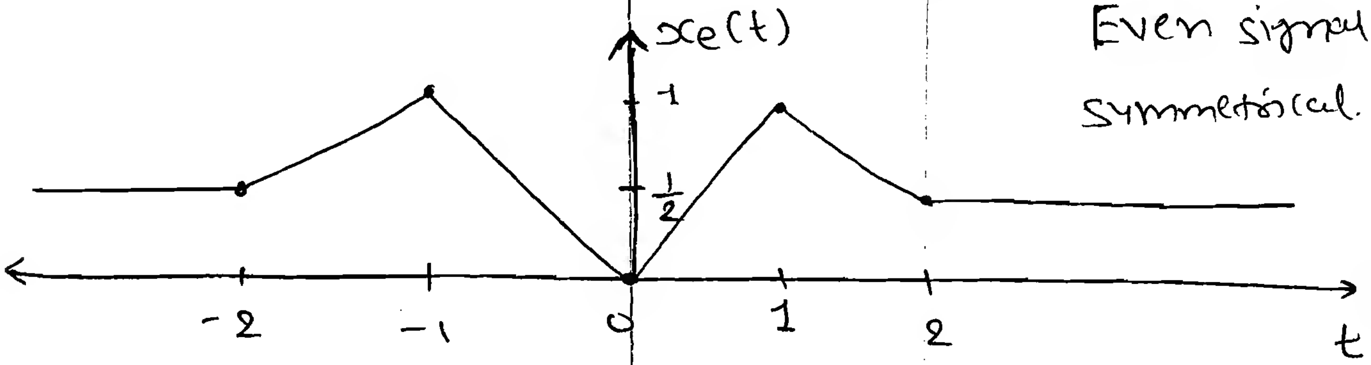
Q Find the odd and even signal for the following signal.

⇒

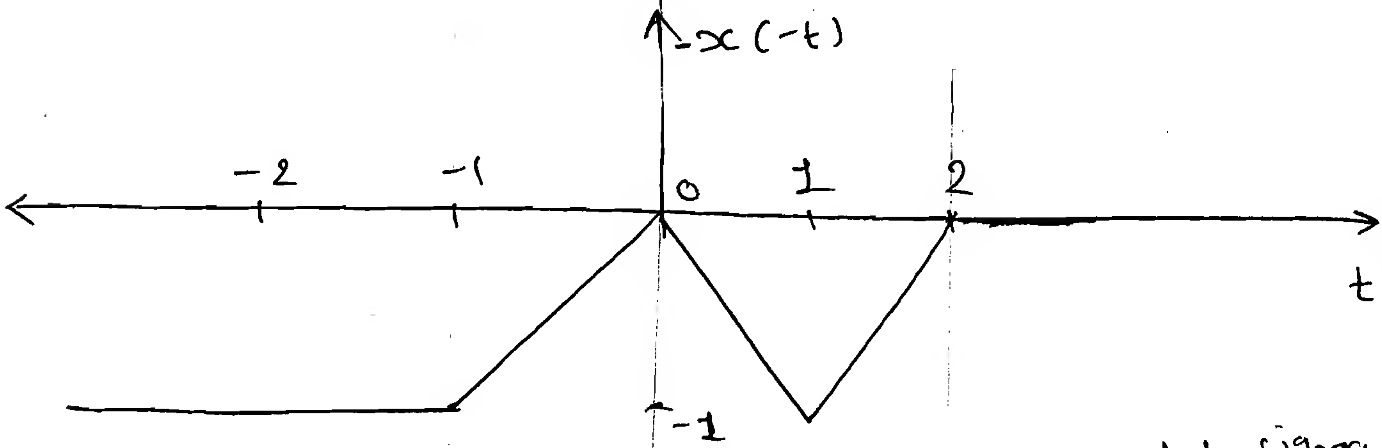


Sol^n:

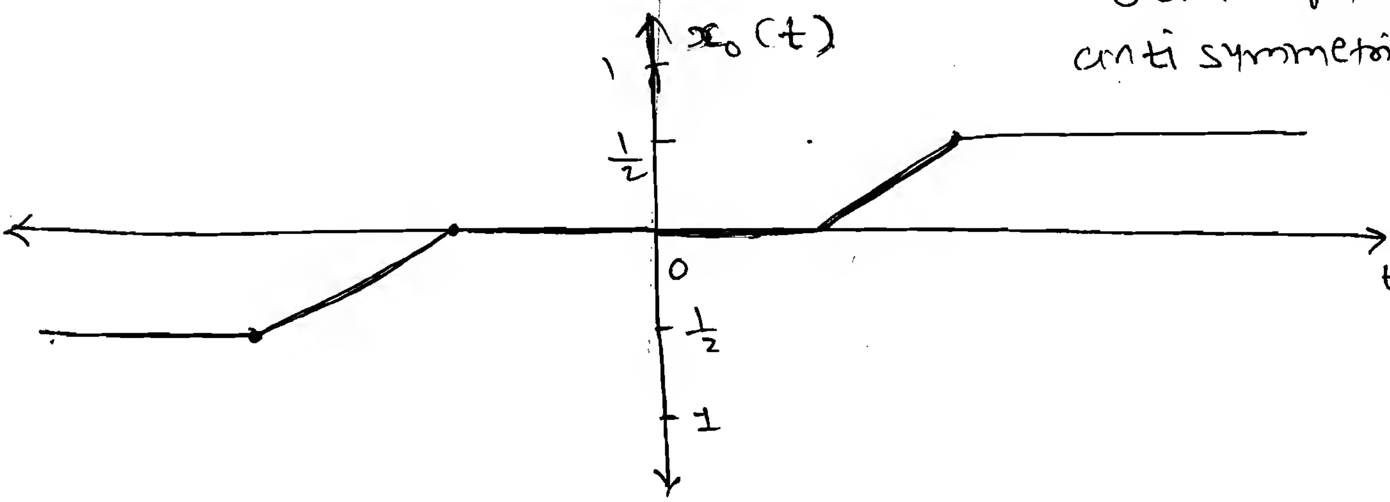




Even signal symmetrical.



odd signal anti symmetrical



⇒ Area under even signal:

$$\int_{-a}^{a} x_e(t)\,dt = 2\int_{0}^{a} x_e(t).dt.$$

⇒ Area under odd signal = zero.

⇒

e + e = o	e . e = e
o + o = o	o . o = o
e + o = neno	e . o = o

$$\Rightarrow \frac{d}{dt}(e) = o \quad \& \quad \frac{d}{dt}(o) = e.$$

$$\Rightarrow \int (e)dt = o \quad \& \quad \int (o)dt = e.$$

⇒ $E_{total} = E_{x(t)} + \cancel{E} = E_{even} + E_{odd}.$

$$E_{x(t)} = \int_{-\infty}^{+\infty} [x_e(t) + x_o(t)]^2 dt.$$

$$= \int_{-\infty}^{\infty} x_e^2(t).\,dt + \int_{-\infty}^{\infty} x_o^2(t).dt + 2\int_{-\infty}^{\infty} \cancelto{0}{x_e(t).x_o(t)dt}$$

$E_{total} = E_{even} + E_{odd}.$

$\boxed{Q}$ The Conjugate Antisymmetric part of

$x[n] = \{ 1+j2, 3, j4 \}$.

$$x[n] = \{ \underset{-1}{\underset{\uparrow}{1+j2}},\ \underset{0}{\underset{\uparrow}{3}},\ \underset{1}{\underset{\uparrow}{j4}} \}$$

Soln:

$$x_{oc}[n] = \frac{x[n] - x^*[-n]}{2}$$

$$x_{oc}[-1] = \frac{x[-1] - x^*[1]}{2}$$

$$= \frac{1+j2 - \{-j4\}}{2}$$

$$x_{oc}[-1] = \frac{1+j6}{2}$$

similarly,

$$x_{oc}[0] = \frac{x[0] - x^*[0]}{2} = \frac{3-3}{2} = 0.$$

$$x_{oc}[1] = \frac{x[1] - x^*[-1]}{2} = \frac{j4 - 1 + j2}{2}$$

$$x_{oc}[1] = \frac{-1+j6}{2}.$$

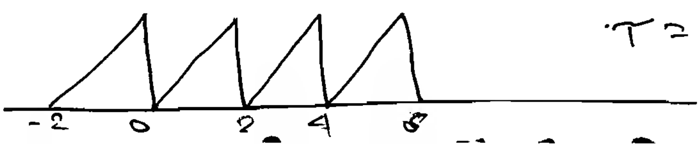
$$\Rightarrow x_{oc}[n] = \left\{ \frac{1+j6}{2}, 0, \frac{-1+j6}{2} \right\}.$$

[3] Periodic & Non-Periodic signal.

$\Rightarrow$ Harmonics : $n\omega_0$

$x(t) = x(t+T)$

$T \rightarrow$ Least period of $x(t)$.



$T = 2$ sec.

* **Steps for finding time period of $x_1(t) + x_2(t) + x_3(t) + \dots$**

① $T_1, T_2, T_3, \dots$

② $T_1/T_2, \; T_1/T_3, \; T_1/T_4, \dots$

③ If the ratios of second step is rational no. then the overall signal is periodic.

④ L.C.M. of Denominators of 2^{nd} step.

⑤ $T = (\text{L.C.M.})\, T_1$.

Q $y_1(t) = \cos 50\pi t + \sin 60\pi t.$

Sol^n: ① $\omega_1 = 50\pi$

$\therefore \dfrac{2\pi}{T_1} = 50\pi$

$T_1 = \dfrac{1}{25}$

$\omega_2 = 60\pi$

$\dfrac{2\pi}{T_2} = 60\pi$

$T_2 = \dfrac{1}{30}$.

② $\dfrac{T_1}{T_2} = \dfrac{30}{25} = 6/5$

③ valid ✓

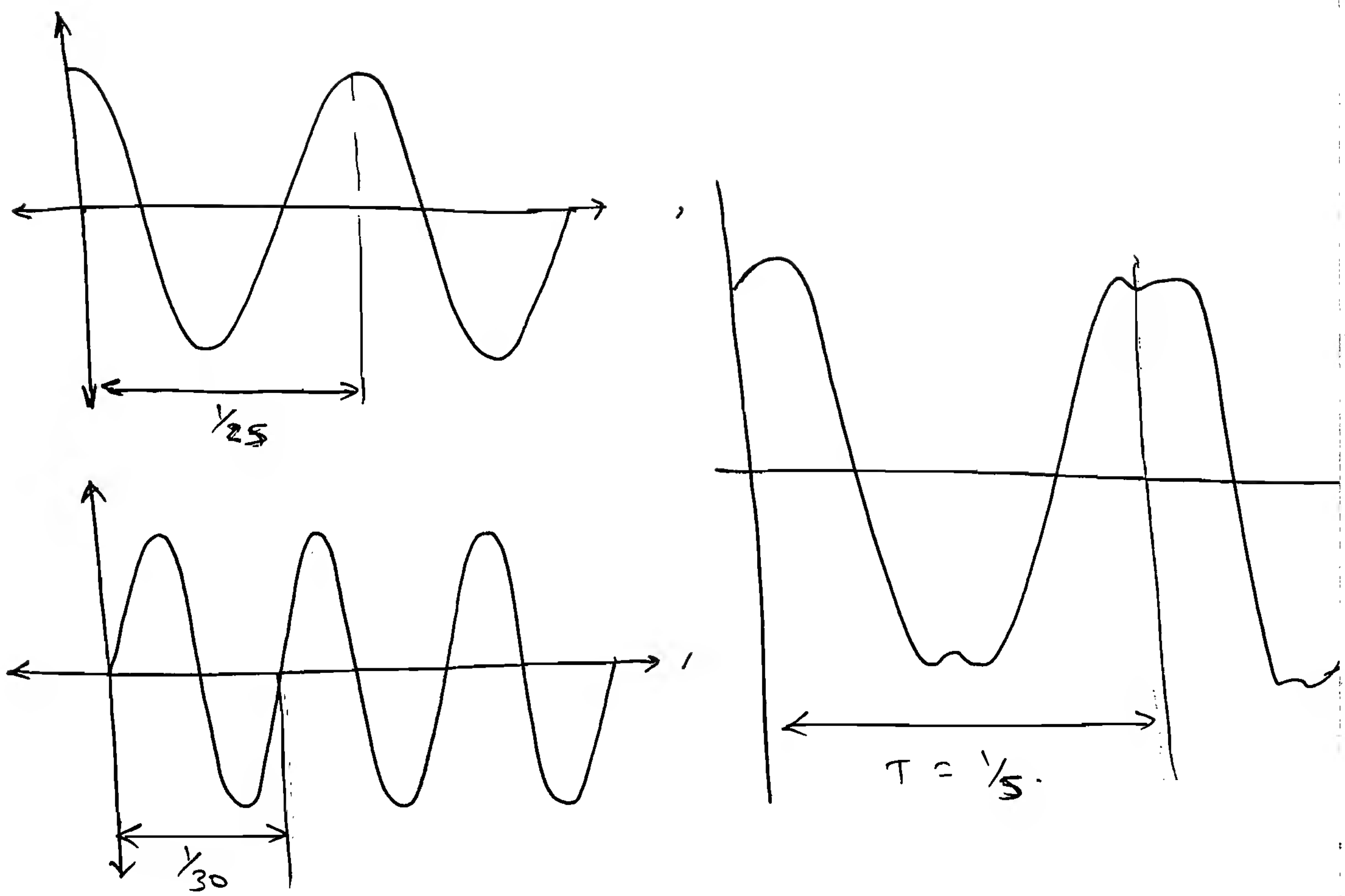
④ L.C.M. $= 5$.

⑤ $T = (\text{L.C.M.}) \times T_1$

$= 5 \times \dfrac{1}{25}$

$$\boxed{T = \frac{1}{5}}$$

$\Rightarrow$



* Alternative.

$\Rightarrow$ G.C.D. (Greatest common factor).

$$50\pi \Rightarrow 5\,(10\pi).$$

$$60\pi \Rightarrow 6\,(10\pi).$$

$$\therefore \quad \omega_0 = 10\pi$$

$$\frac{2\pi}{T} = 10\pi \Rightarrow \boxed{T = \frac{1}{5}.}$$

③ $y(t) = e^{j\frac{5\pi t}{6}} + e^{j\frac{\pi t}{3}}.$

Solⁿ:

$$y(t) = e^{j\frac{5\pi t}{6}} + e^{j\frac{2\pi t}{6}}.$$

$$\therefore \quad \text{GCD}\left(\frac{5\pi}{6}, \frac{2\pi}{6}\right).$$

$$= \frac{\pi}{6} = \omega_0 \Rightarrow \frac{2\pi}{T} = \frac{\pi}{6} \Rightarrow \boxed{T = 12}$$

④ $y(t) = \sin\left(\frac{2\pi t}{3}\right) \cdot \cos\left(\frac{4\pi t}{5}\right)$.

$\underline{\underline{\text{Sol}^n}}$:

$y(t) = \frac{1}{2} \cdot 2\sin\left(\frac{2\pi t}{3}\right) \cdot \cos\left(\frac{4\pi t}{5}\right)$.

$y(t) = \frac{1}{2}\sin\left(\frac{22\pi t}{15}\right) - \frac{1}{2}\sin\left(\frac{2\pi}{15}\right)$.

$\therefore \quad GCD\left(\frac{22\pi}{15}, \frac{2\pi}{15}\right)$.

$= \frac{2\pi}{15} = \omega_0$

$\Rightarrow \frac{2\pi}{T} = \frac{2\pi}{15}$

$\Rightarrow \boxed{T = 15}$

⑤ $y(t) = y_1(t) + y_2(t) + y_3(t)$.

$\downarrow \quad \downarrow \quad \downarrow$

$T_1 = 1.08 \quad T_2 = 3.6 \quad T_3 = 2.205$.

⑥ $x(t) = \cos 13\pi t + \sin 17t$.

$\underline{\underline{\text{Sol}^n}}$: $T_1 = \frac{2}{13}, \quad T_2 = \frac{2\pi}{17}$.

So, $\frac{T_1}{T_2} = \frac{17}{13\pi}$

$\uparrow$

irrational

So, Non-periodic.

⑦ $y(t) = je^{j10t}$.

$\underline{\underline{\text{Sol}^n}}$: $y(t) = e^{j\frac{\pi}{2}} \cdot e^{j10t} = e^{j\left(\frac{\pi}{2}+10t\right)}$.

$\Rightarrow$ Additional phase angle of $90°$

$\Rightarrow \quad j = e^{j\pi/2}$

$\Rightarrow$ It never changes periodicity.

$\omega_0 = 10 \Rightarrow T = \frac{2\pi}{\omega_0} = \pi/5.$

Proof: $y\left(t+\frac{\pi}{5}\right) = j \cdot e^{j\left(10t+\frac{\pi}{5}\right)}$

$= j \cdot e^{j10t} \cdot e^{j2\pi}$

$= j \cdot e^{j10t} \cdot (\cos 2\pi + j\sin 2\pi)$

$= j \cdot e^{j10t}$

$\therefore y\left(t+\frac{\pi}{5}\right) = y(t)$

(7) $x(t) = e^{-(2+j3)t}$

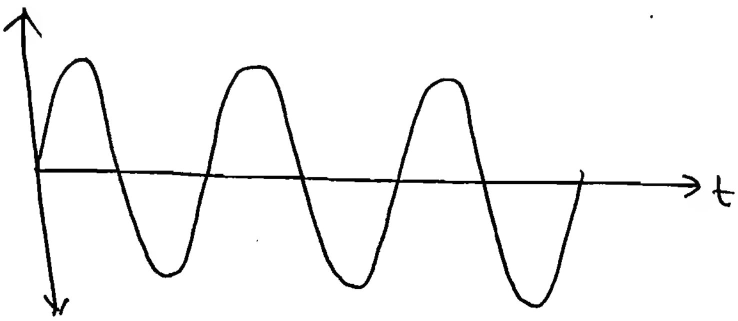
Sol^n:

$x(t) = e^{-2t} \cdot e^{j3t}$

Non-periodic periodic.

(8) $y(t) = \sin\left(\frac{\pi t}{6}\right) \cdot u(t).$

Sol^n:



$\Rightarrow$ Non-Periodic

$\sin\left(\frac{\pi}{6}\right)t \cdot u(t).$

All periodic signals are everlasting signal $\Rightarrow -\infty$ to ∞ time

⑨ $x(t) = Ev\{\cos 3\pi t \cdot u(t)\}$.

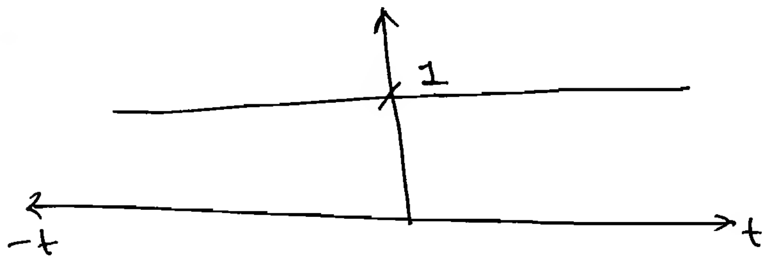
Sol^n:

$$x_e(t) = \frac{\cos 3\pi t \cdot u(t) + \cos 3\pi(-t) \cdot u(-t)}{2}$$

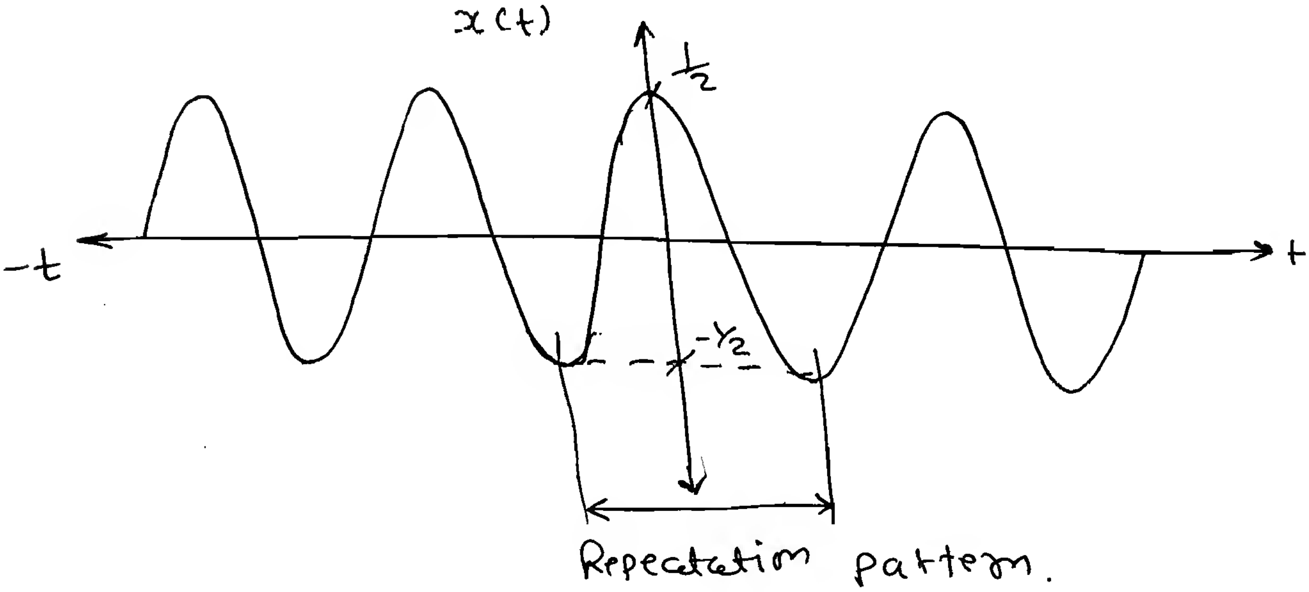
$$= \frac{\cos 3\pi t \cdot u(t) + \cos 3\pi t \cdot u(-t)}{2}$$

$$\therefore \quad x(t) = \frac{\cos 3\pi t}{2} \left[u(t) + u(-t) \right].$$

$u(t) + u(-t) \Rightarrow$



$\Rightarrow$



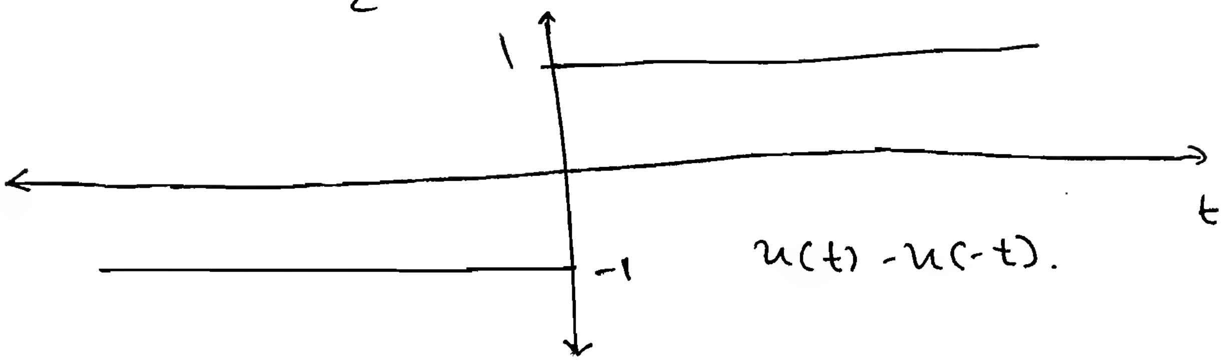
$$\Rightarrow \quad \omega_0 = \frac{2\pi}{T} = 3\pi$$

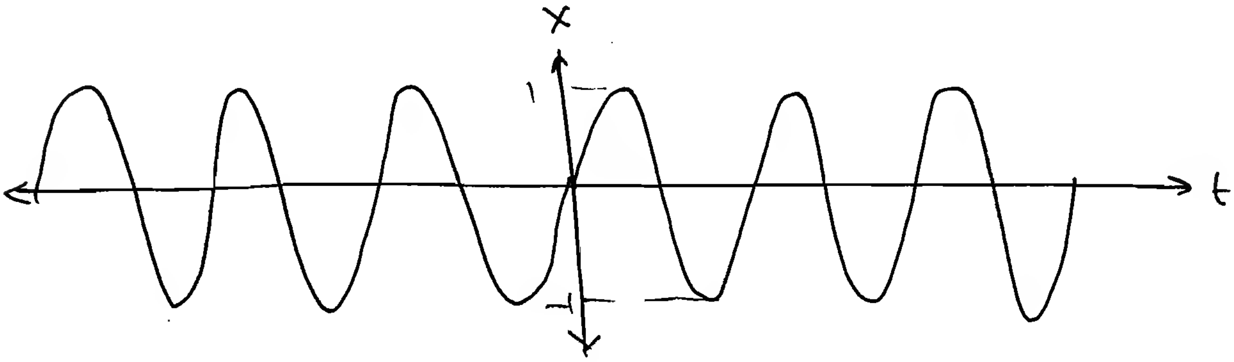
$$\Rightarrow \boxed{T = \frac{2}{3}}$$

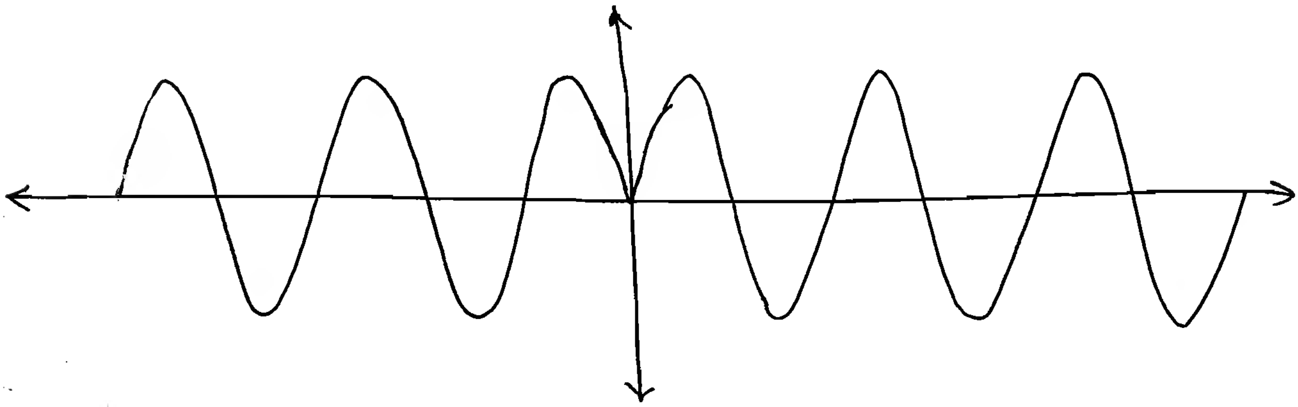
* $x(t) = Ev\{\sin 3\pi t \cdot u(t)\}$.

Sol^n: $$x(t) = \frac{\sin 3\pi t \cdot u(t) + \sin 3\pi(-t) \cdot u(-t)}{2}$$

$\Rightarrow \quad x(t) = \dfrac{\sin 3\pi t}{2} \quad [u(t) - u(-t)].$



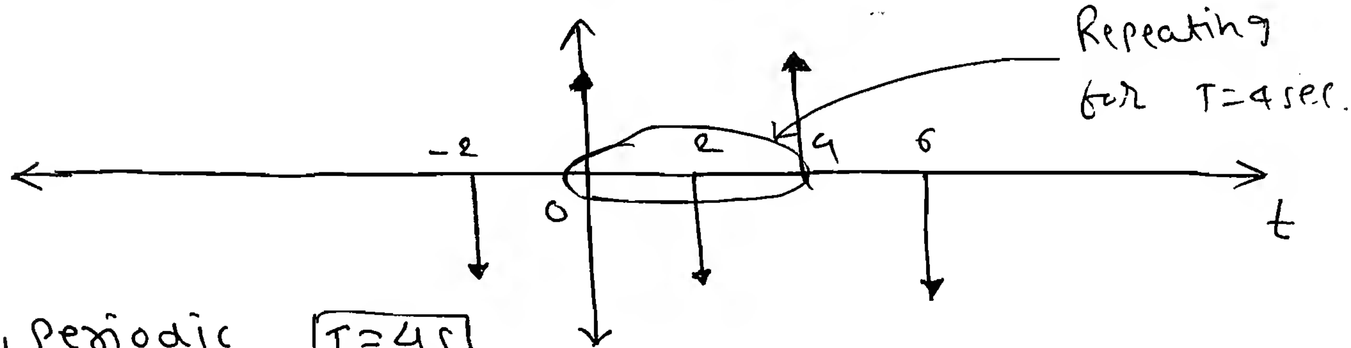




So, Non-periodic.

(11) $$y(t) = \sum_{k=-\infty}^{\infty} (-1)^k \cdot \delta(t-2k).$$

$\underline{\underline{Sol^n}}$: $y(t) = \cdots + \delta(t+2) + \delta(t) - \delta(t-2) + \delta(t-4) + \cdots$



So, Periodic $\boxed{T=4s}$

☆ $x[n] = \sin\left[\frac{3\pi n}{5}\right]$.

⇒ Continuous sinusoids and Complex sinusoids are periodic for every value of ω_0 whereas the equivalent discrete terms are periodic if $\frac{\omega_0}{2\pi}$ is a Rational Number $\left(\frac{m}{N}\right)$.

⇒ To make N as the time period signal, m no. of full cycles continuous signal is repeated.

$$\Rightarrow \quad x[n] = x[n+N].$$

$$x[n] = \sin\omega_0 n.$$

$$\Rightarrow x[n+N] = \sin\omega_0(n+N).$$

$$= \sin.\omega_0 n.\underbrace{\cos\omega_0 N}_{1} + \cos\omega_0 n.\underbrace{\sin\omega_0 N}_{0}.$$

$$\Rightarrow \cos\omega_0 N = \cos 2\pi m = 1. \quad \text{for } m = 0, \pm 1, \pm 2, \ldots$$

$$\therefore \omega_0 N = 2\pi m.$$

$$\therefore \boxed{\frac{\omega_0}{2\pi} = \frac{m}{N}.}$$

→ Here, $\omega_0 = \frac{3\pi}{5}$.

$\therefore \frac{\omega_0}{2\pi} = \frac{3}{10} = \frac{m}{N}$

$\therefore$ Time period of Discrete signal $\boxed{N=10}$

② $x[n] = \cos\left[\frac{n}{6} + \frac{\pi}{4}\right]$.

Sol^n:

here, $\omega_0 = \frac{1}{6}$.

$\therefore \frac{\omega_0}{2\pi} = \frac{1}{12\pi}$, so irrational

$\therefore$ Non-periodic.

③ $x[n] = \sin\left(\frac{\pi n}{3}\right) + \cos\left(\frac{\pi n}{4}\right)$.

Sol^n:

here, $\omega_{01} = \frac{\pi}{3}$, $\omega_{02} = \frac{\pi}{4}$.

$\therefore N_1 = \cancel{\tfrac{1}{6}}\ 6$, $N_2 = 8$.

$\therefore \boxed{N = LCM(N_1, N_2)}$ * * *

$\boxed{\begin{array}{l} + \rightarrow LCM \\ \div \rightarrow GCD. \end{array}}$

$\Rightarrow N = LCM(6, 8)$.

$\boxed{N = 24}$.

$$\begin{array}{r|l} 2 & 6, 8 \\ \hline & 3, 4 \end{array}$$

So, Lcm = $2 \times 3 \times 4$

$= 24$.

(4) $x(t) = 2\cos(150\pi t + 45^\circ)$, $f_s = 200$ Hz

Sol^n: Convert cont^n to discrete by put $t = nT_s$.

$\therefore x[nT_s] = 2\cos(150\pi n \cdot T_s + 45^\circ)$.

$= 2\cos(150 \cdot \pi \cdot n \cdot \frac{1}{f_s} + 45^\circ)$.

$\therefore x[nT_s] = 2\cos(150 \cdot \pi \cdot n \times \frac{1}{200} + 45^\circ)$.

$\Rightarrow \omega_0 = \frac{150\pi}{200}$.

$\therefore \cancel{2\pi} \quad \frac{\omega_0}{2\pi} = \frac{15}{20} \times \cancel{\pi} \times \frac{1}{2\cancel{\pi}} = 3/8$.

So, $\boxed{N = 8}$

(5) $x[n] = (j)^{n/2}$.

Sol^n: $x[n] = e^{j\frac{\pi}{2} \times \frac{n}{2}}$. $\quad (\because j = e^{j\frac{\pi}{2}})$.

$\therefore x[n] = e^{j\frac{n\pi}{4}}$.

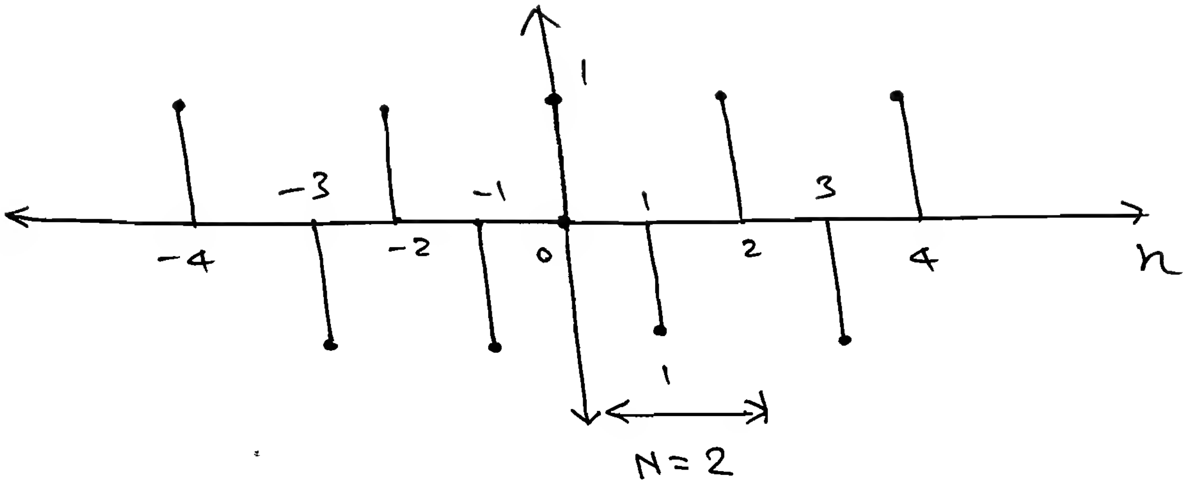
$\therefore \omega_0 = \frac{\pi}{4}$.

$\therefore \frac{\omega_0}{2\pi} = \frac{\pi}{4} \times \frac{1}{2\pi} = \frac{1}{8}$.

So, $\boxed{N = 8.}$

(6) $x[n] = (-1)^n$

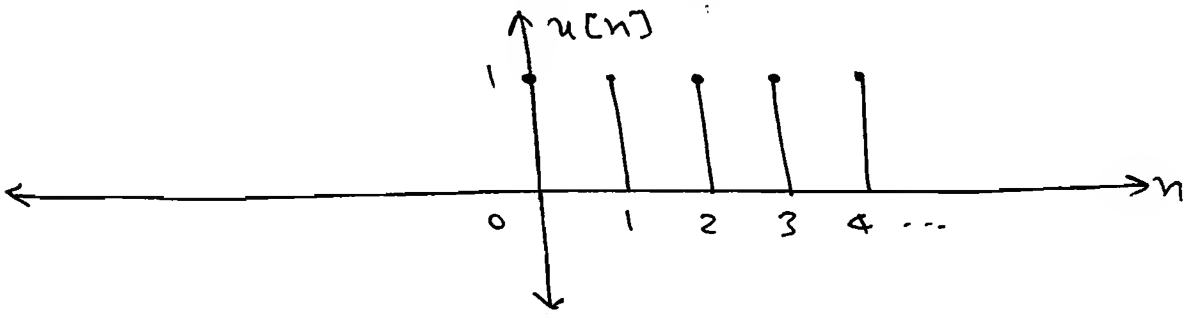
Sol^n:



So, $\boxed{N=2}$

(7) $x[n] = u[n] + u[-n]$

Sol^n:



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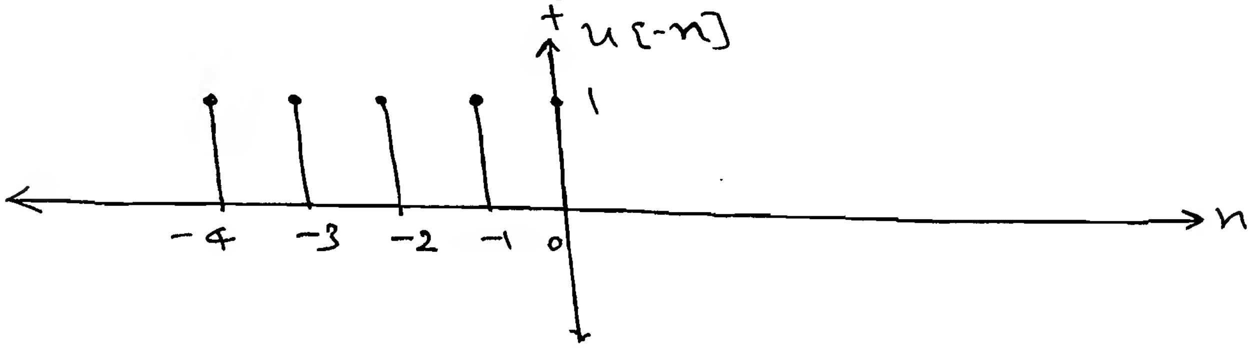

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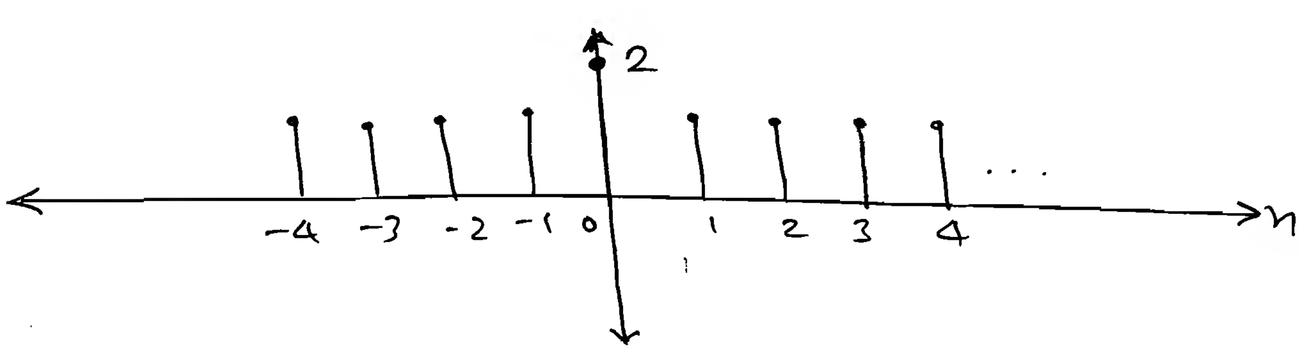
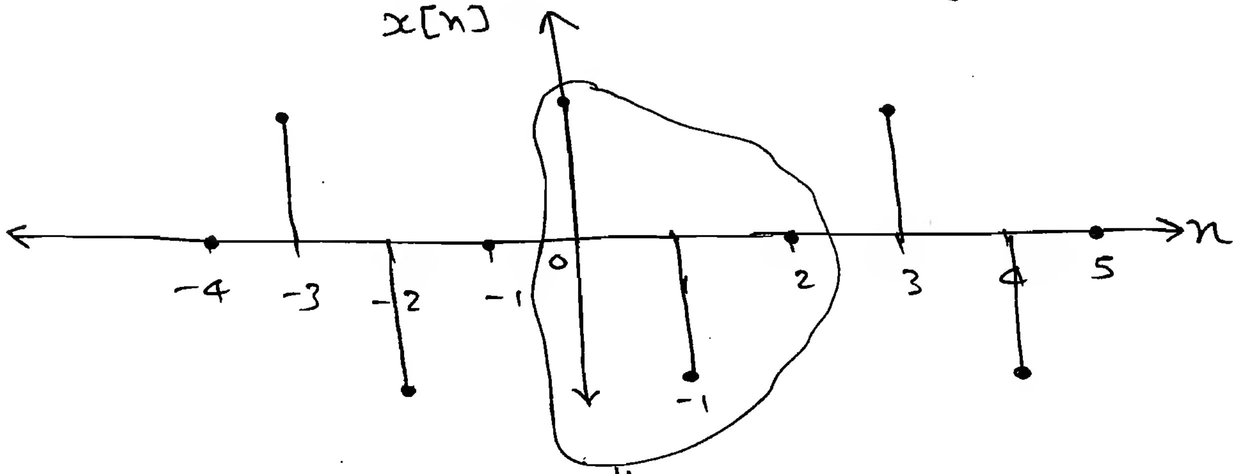


So, non - Periodic.

[8] $x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-3k] - \delta[n-1-3k]$.

Sol^n: $x[n] = \dots + \delta[n+3] - \delta[n+2] + \delta[n] - \delta[n-1] + \delta[n-3] - \delta[n-4] + \dots$



$\Downarrow$

$\boxed{N=3}$

[4] Causal & Non- Causal signals:

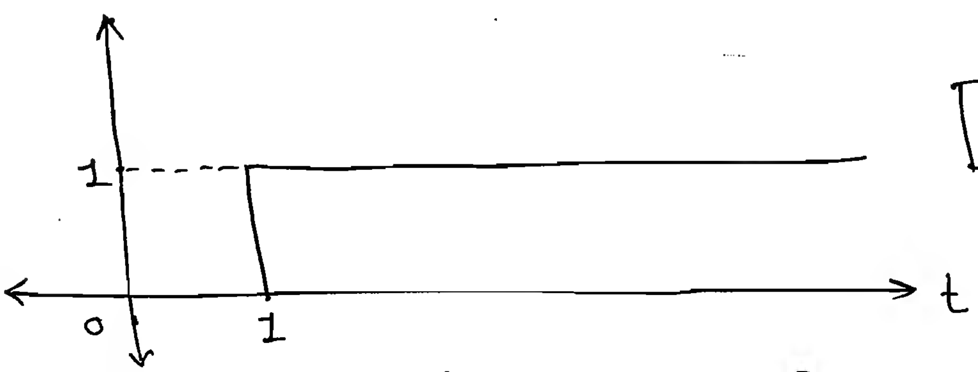
Sol^n: Causal: Defined for +ve time only.

e.g. $\Rightarrow$ $t > 0$ (or) $n \geq 0$.

$\Rightarrow$ $x(t) = 0$; $t < 0$

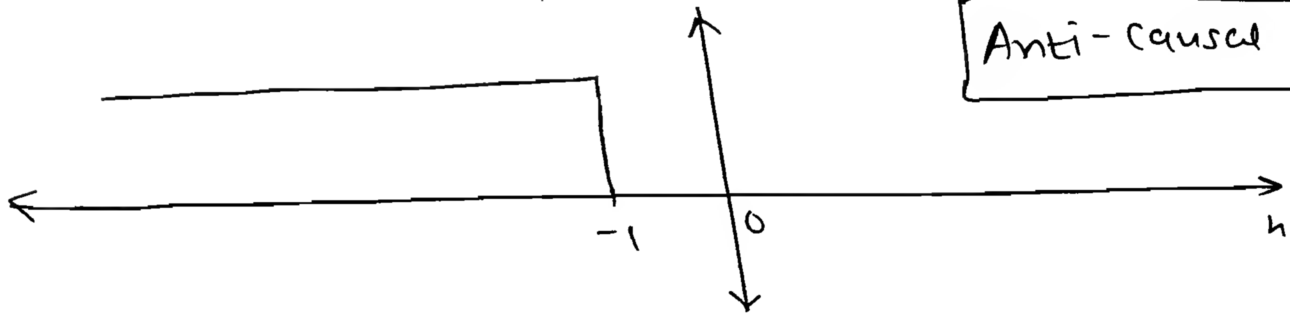
$x[n] = 0$; $n < 0$.

e.g. ① $x(t) = u(t-1)$.



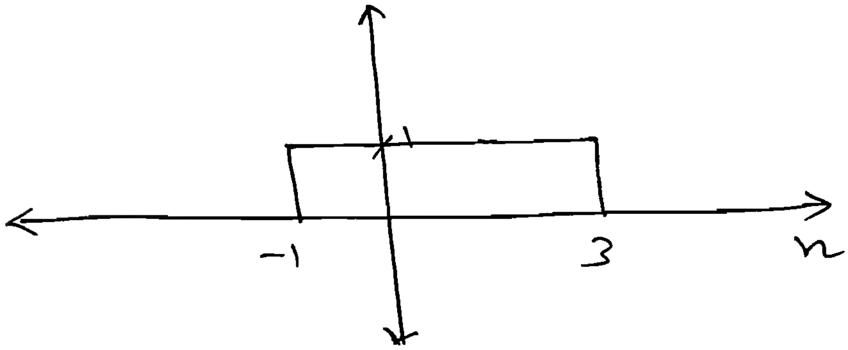
$\boxed{\text{Causal.}}$

(2) $u[-t-1] = u[-(t+1)]$.



Anti-causal

(3)

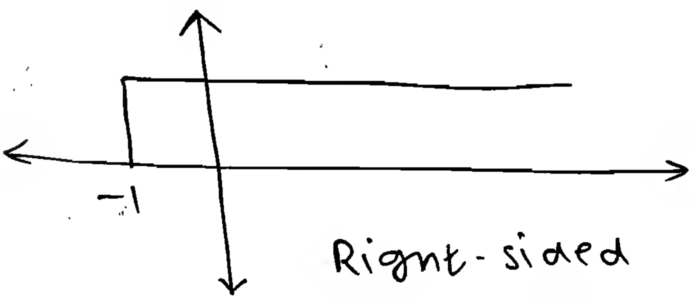


Non-Causal

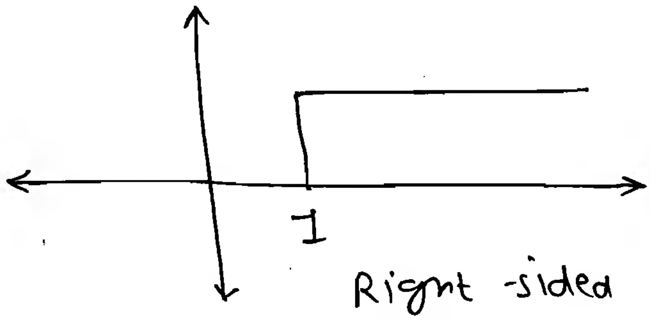
<u>Note:</u>

⇒ All causal signals are Right sided signal, but all right sided signal may (or) may not be causal signal.

⇒ <u>Right</u> - <u>sided:</u> $(t \geq t_0)$

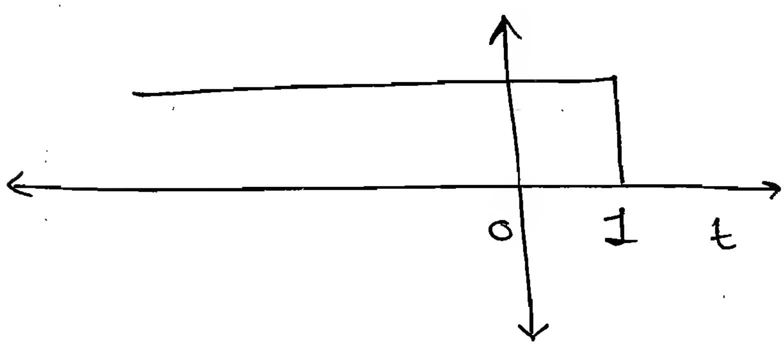


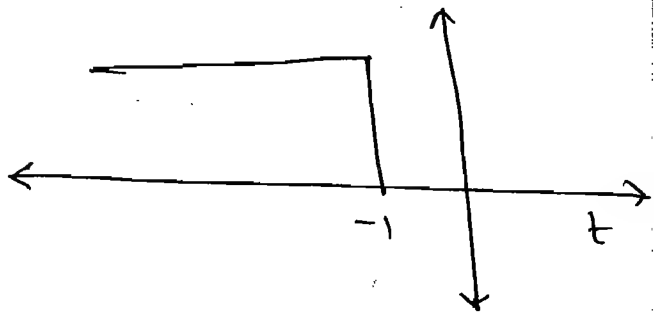
Right-sided
Non-Causal



Right-sided
Causal.

⇒ <u>Left</u> - <u>sided:</u> $(t \leq t_0)$.





→ Periodic and Random signals are Power signal.

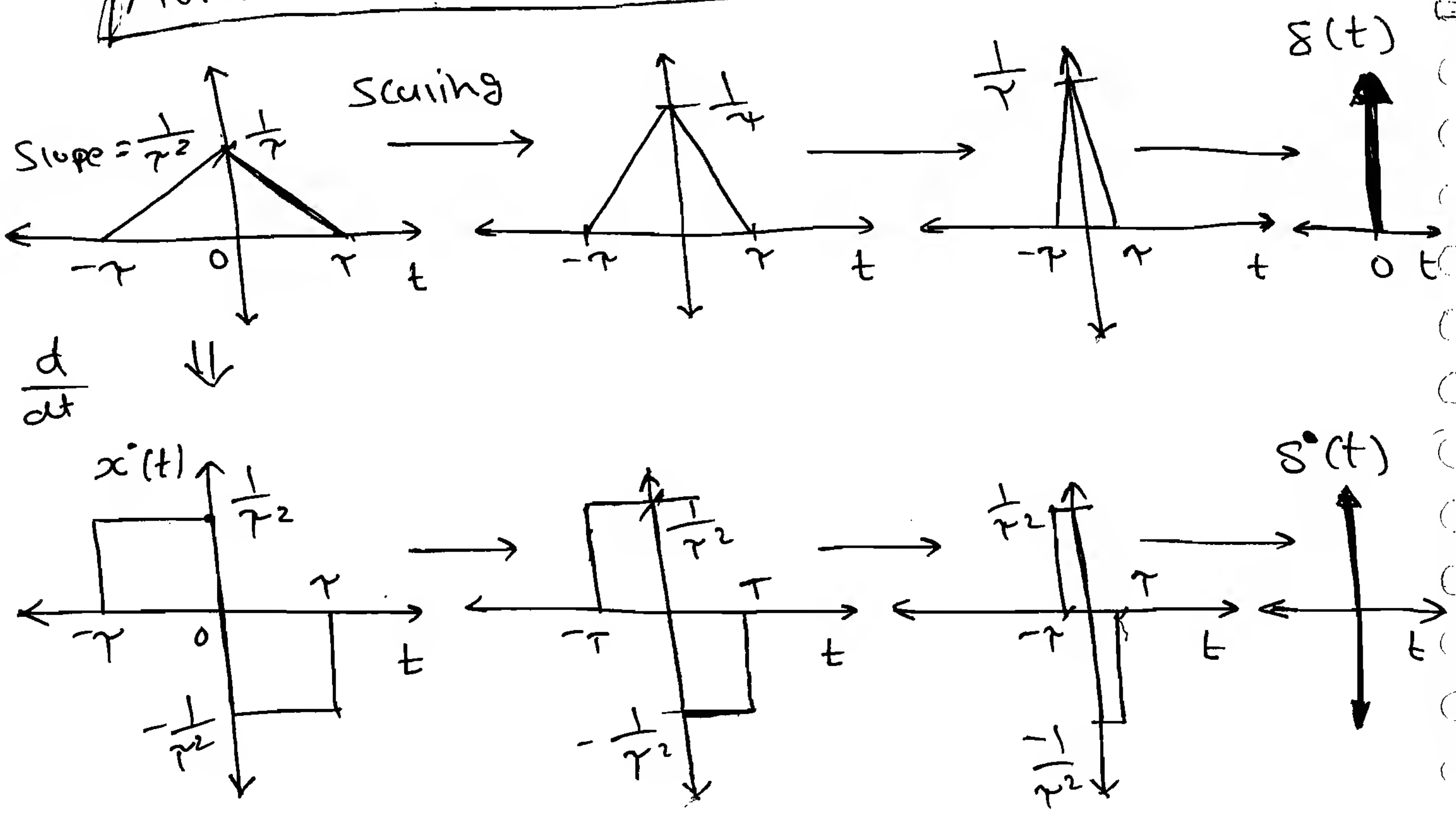
→ But Aperiodic and Deterministic signal may (or) may not be energy signal.

e.g. $u(t)$ → is non-periodic and it is power-signal.

* Doublet:

⇒ By approximating a unit area triangular function we can generate unit impulse fn. In a similar manner by approximating derivative of triangular we can produce doublet. ($\delta'(t)$).

⇒ $\boxed{\text{Area under Doublet is zero.}}$



⇒ Sifting Property:

$$\Rightarrow \int_{t_1}^{t_2} x(t)\cdot \delta(t-t_0)\,dt = x(t_0)\ ; \quad t_1 \le t_0 \le t_2.$$

$$\Rightarrow \boxed{\int_{t_1}^{t_2} x(t)\cdot \delta^{(n)}(t-t_0)\,dt = (-1)^n \left.\frac{d^n x(t)}{dt^n}\right|_{t=t_0}}$$

⇒ Derivative of Doublet is Triplet and we should consider parabolic curve instead of triangular curve since triplet is double derivative of impulse. Double derivative of Parabolic curve is unit step function.

[a] $\int_0^{\infty} e^{-2t^2}\cdot \delta'(t-10)\,dt$.

Sol^n: $t_0 = 10$, $0 < 10 < \infty$.

So, $= (-1)^n \cdot \left.\frac{d^n}{dt} x(t)\right|_{t=t_0}$

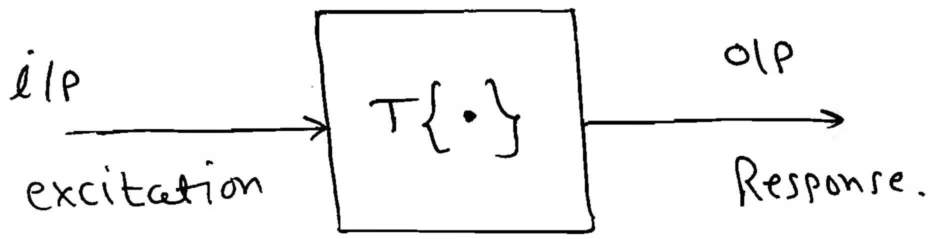
$= (-1)^1 \frac{d}{dt}\cdot \left. e^{-2t^2}\right|_{t=10}$

$= -\left. e^{-2t^2}\cdot(-4t)\right|_{t=10}$

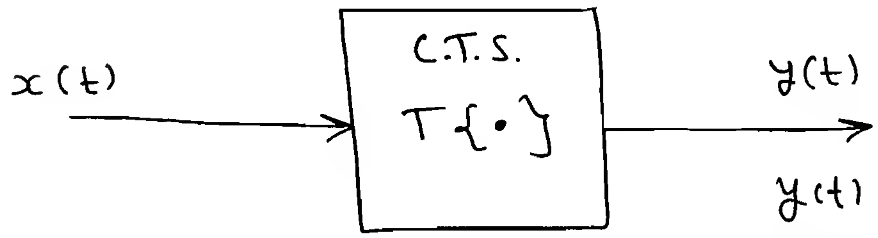
$= 40\, e^{-200}$

☆ Systems :-

⇒



⇒



$y(t) = T\{x(t)\}$.

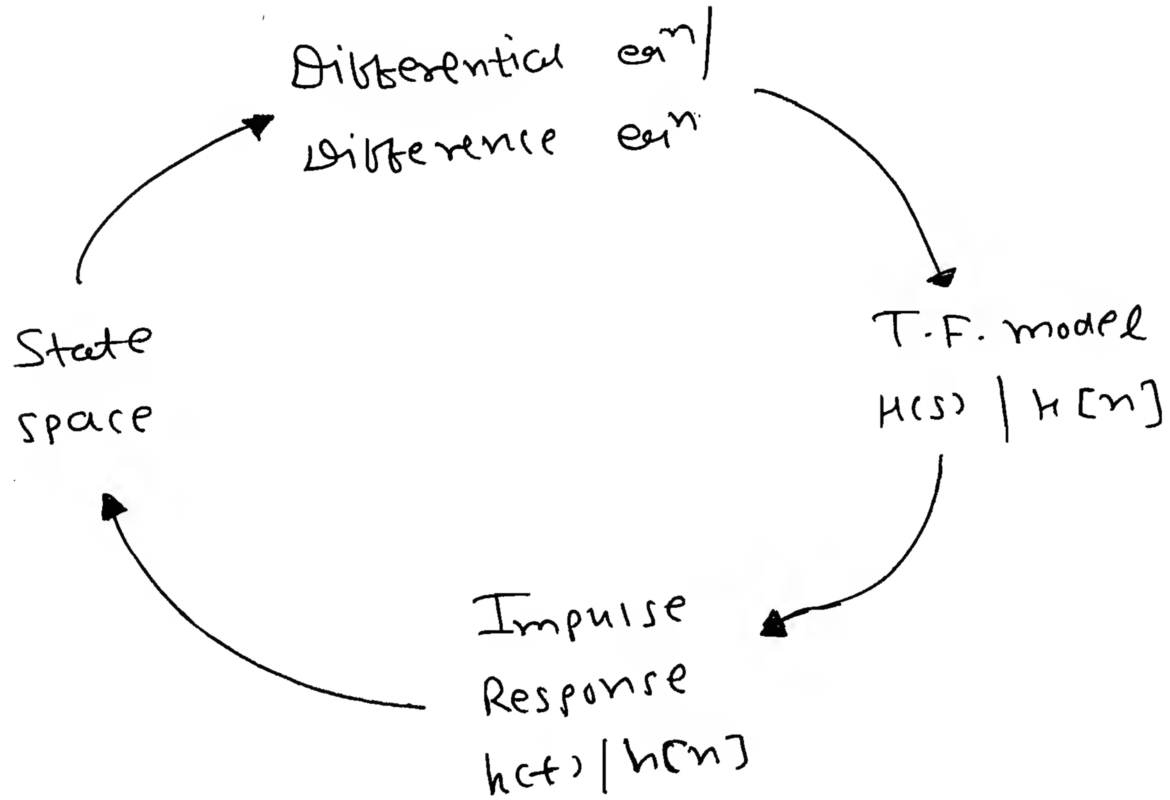
e.g. $y(t) = \ln x(t)$

$= x^2(t)$.

⇒ Designing systems ⇒ Synthesis.

⇒ ~~An~~ Getting o/p ⇒ Analysis.

⇒ –



$\Rightarrow$ Linear, Stable, Invertible $\rightarrow$ Considering Amp.

$\Rightarrow$ T.I., static, Causal $\Rightarrow$ Time.

[1] Linear & Non-linear System:

$\Rightarrow$ Linear $\Rightarrow$ Supperpossion = Additivity + Homoginity.

① Additivity:

$$x_1(t) \xrightarrow{T} y_1(t)$$

$$x_2(t) \xrightarrow{T} y_2(t).$$

$$\Rightarrow \quad x_1(t) + x_2(t) \xrightarrow{T} y_1(t) + y_2(t).$$

② Scaling (Homoginity):

$$\Rightarrow \quad \alpha x(t) \longrightarrow \alpha y(t).$$

(α = need not be real; can be Imaginary).

e.g. $y(t) = x(t) \cdot x(t-3)$.

$$x_1(t) \longrightarrow y_1(t) = x_1(t) \cdot x_1(t-3).$$

$$x_2(t) \longrightarrow y_2(t) = x_2(t) \cdot x_2(t-3).$$

$$y(t) = (x_1(t) + x_2(t)) \longrightarrow (x_1(t) + x_2(t)).(x_1(t-3) + x_2(t-3)).$$

$\Rightarrow$ $\boxed{\text{Non-linear}}$ $\qquad \neq y_1(t) + y_2(t)$

$\boxed{a}$ - ① $y(t) = x(t) \cdot \cos 5t$.

Sol^n:

$x_1(t) \Rightarrow y_1(t) = x_1(t) \cdot \cos 5t$.

$x_2(t) \Rightarrow y_2(t) = x_2(t) \cdot \cos 5t$.

$\Rightarrow x_1(t) + x_2(t) \Rightarrow y(t) = (x_1(t) + x_2(t)) \cos 5t$

$= x_1(t) \cdot \cos 5t + x_2(t) \cdot \cos 5t$

$y(t) = y_1(t) + y_2(t)$

$\Rightarrow$ Linear.

NOTE:

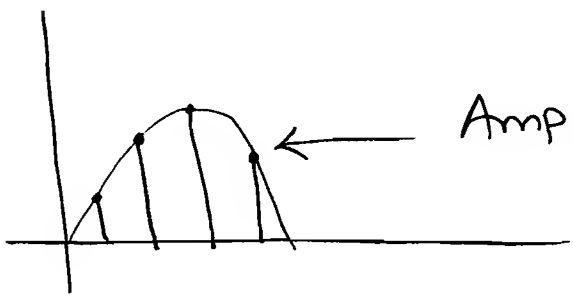
$\Rightarrow$ Unknown signal Product make the system non-linear.

② $y(t) = \int_{-\infty}^{t} x(\tau) \cdot d\tau$.

$\Rightarrow$ Linear.

③ $y(t) =$ Sample $\{x(t)\}$.

$\Rightarrow$ System operation is sampling.



Amplitude is not changing so, Linear.

$\Rightarrow$ Sampling is a linear operation, but time dependent (T_s) so, time variant.

③ $y(t) = \sin\{x(t)\}$.

$\downarrow$

Non - linear

Sol^n: $y_1(t) = \sin\{x_1(t)\}$, $y_2(t) = \sin\{x_2(t)\}$.

$y(t) \Rightarrow \sin\{x_1(t) + x_2(t)\}$

$\neq \sin\{x_1(t)\} + \sin\{x_2(t)\}$.

Note: When o/p is trignomatric function System is always Non-Linear.

④ $y(t) = |m(t)|$

Sol^n: Non - linear $\Rightarrow$ All +ve values becomes -ve values

o/p due to $-5x(t) = |-5x(t)| = 5|x(t)|$.

$\alpha y(t) = -5|x(t)| \neq 5|x(t)|$

So, Non-linear.

Note: Modulas of i/p makes the system Non - linear. because all -ve values becomes +ve value.

⑤ $y(t) = 3x(t) + 5$..

Sol^n: Non-linear.

o/p due to $\alpha y(t) = \alpha(3x(t) + 5)$.

$= 3\alpha x(t) + 5\alpha$.

o/p due to $\alpha x(t)$

$= 3\alpha x(t) + 5.$

So, Non-linear.

<u>Note</u>: Addition of Constant makes the System Non-linear.

→ This system is also called as "Incremental Linear" since o/p is increasing because of constant term.

[6] $y[n] = 2^{x[n]}.$

<u>Soln</u>: $y_1[n] = 2^{x_1[n]}, \quad y_2[n] = 2^{x_2[n]}.$

∴ o/p due to $x_1[n] + x_2[n]$

$$= 2^{x_1[n]} \cdot 2^{x_2[n]}$$

$$\neq 2^{x_1[n]} + 2^{x_2[n]}.$$

So, Non-linear.

<u>Note</u>: Exponential terms makes the system Non-Linear.

[7] $y[n] = \text{sgn}[x[n]].$

<u>Soln</u>: $y[n] = 1; \quad x[n] > 0.$

$= -1; \quad x[n] < 0.$

So, Non-linear.

⇒ For signam we are always maintaing Const. amp of ± 1. so it is non-linear.

Note: When o/p signal is input dependent then it is always Non-Linear.

[8] $y[n] = x^*[n]$.

Sol^n: o/p due to $(2+j3) = \{(2+j3)\, x[n]\}^*$.

$$= (2-j3) \cdot x[n]$$

$$\alpha\, x[n] = (2+j3) \cdot x[n] \neq \text{Non-linear.}$$

[9] $y[n] = \text{Real}\,[x(n)]$.

Sol^n: o/p due to $2+j3 = \text{Real}\,\{(2+j3) \cdot x[n]\}$.

$$= 2 \cdot \text{Real}\,[x[n]].$$

$$(2+j3)\,\text{Real}\,\{x[n]\} \neq 2\,\text{Real}\,(x(n)).$$

So, Non-linear.

Note: → Real, Complex part and Conjugate all are Non-linear.

[10] $y[n] = \text{median}\,[x(n)]$.

Sol^n: o/p$_1$ = $\{1, 2, 3, 4, 5\}$ = 3. $\quad$ $3+1$ $\quad$ 4.

o/p$_2$ = $\{1, 1, 2, 1, 2\}$ = 1.

$OP_1 + OP_2 = \{1, 1, 1, 1, 2, 2, 2, 3, 4, 5\}.$

$$\frac{2+2}{2} = 2.$$

So, Non-linear.

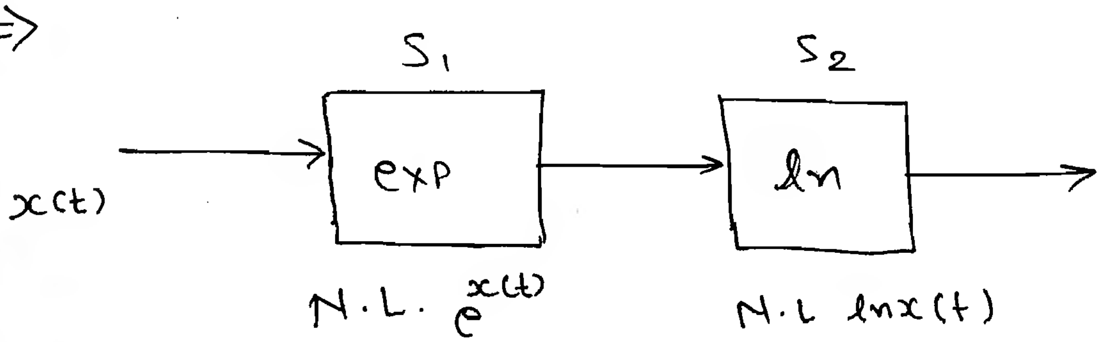
<u>Note</u>: → Quantization is Non-linear ope. &

Sampling is Linear operation..

→ Phase - plane Analysis → Non-linear.

* <u>Two - System Connected in Series.</u>

⇒

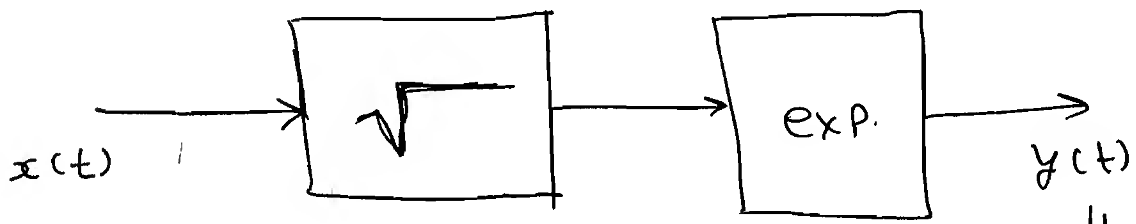


$y(t) = x(t).$

⇓

Linear.

⇒



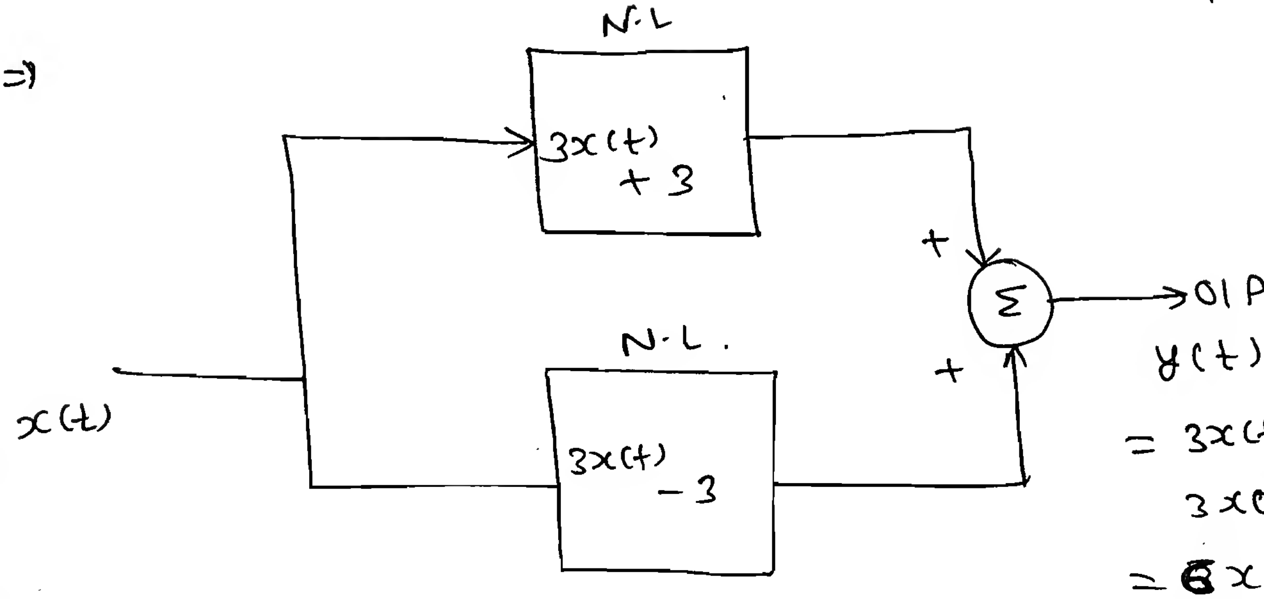
⇓

Non-linear.

⇒ When both S_1 & S_2 are opposite to each other then it becomes Linear.

* Two - Systems Connected in Parallel:

=>

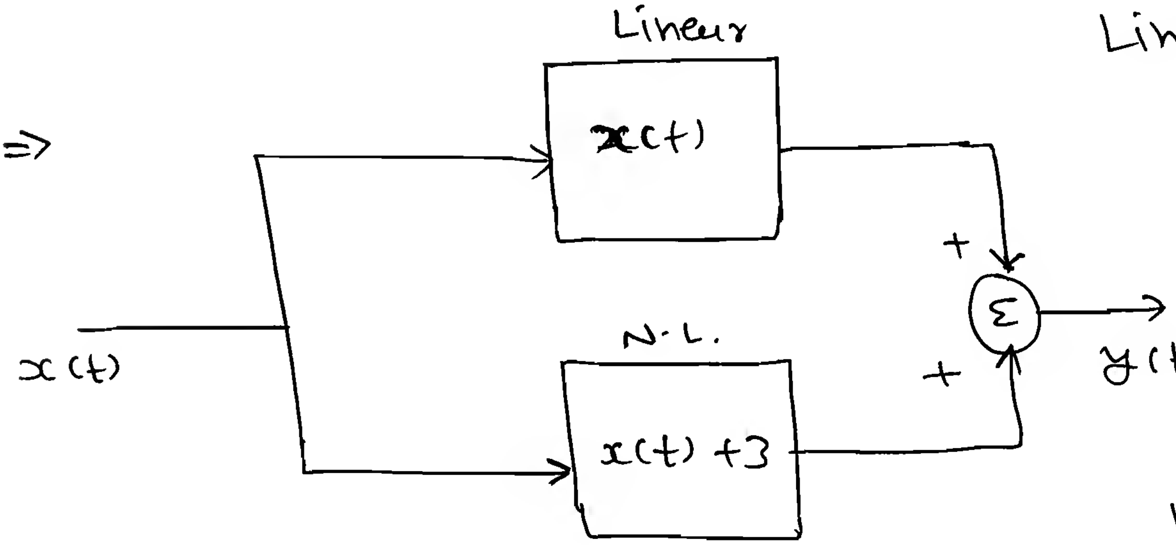


$= 3x(t) + 3$
$\;\; 3x(t) - 3$
$= 6x(t)$

⇓

Linear.

=>



$y(t) = 2x(t) + 3$

⇓

Non-linear.

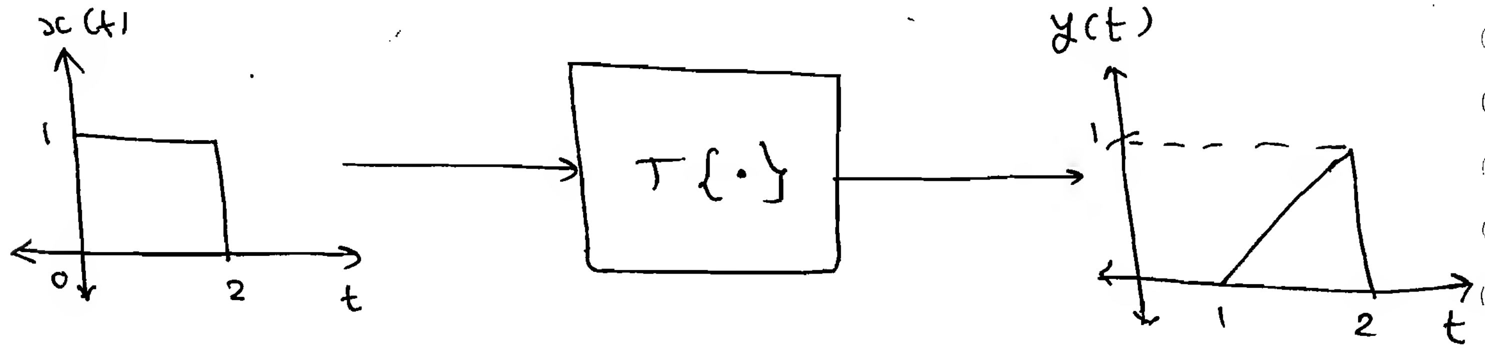
[2] Time Invariant | Shift Invariant :-

=> A system is T.I. if I/P O/P char. do not change with time.

=> If $x(t) \longrightarrow y(t)$

$x(t - t_0) \longrightarrow y(t - t_0)$.

T.I. $\quad y(t)\Big|_{t=t_0} = y(t)\Big|_{t=t-t_0}$.



e.g.

$\boxed{a}$ ① $y(t) = t \cdot x(t)$.

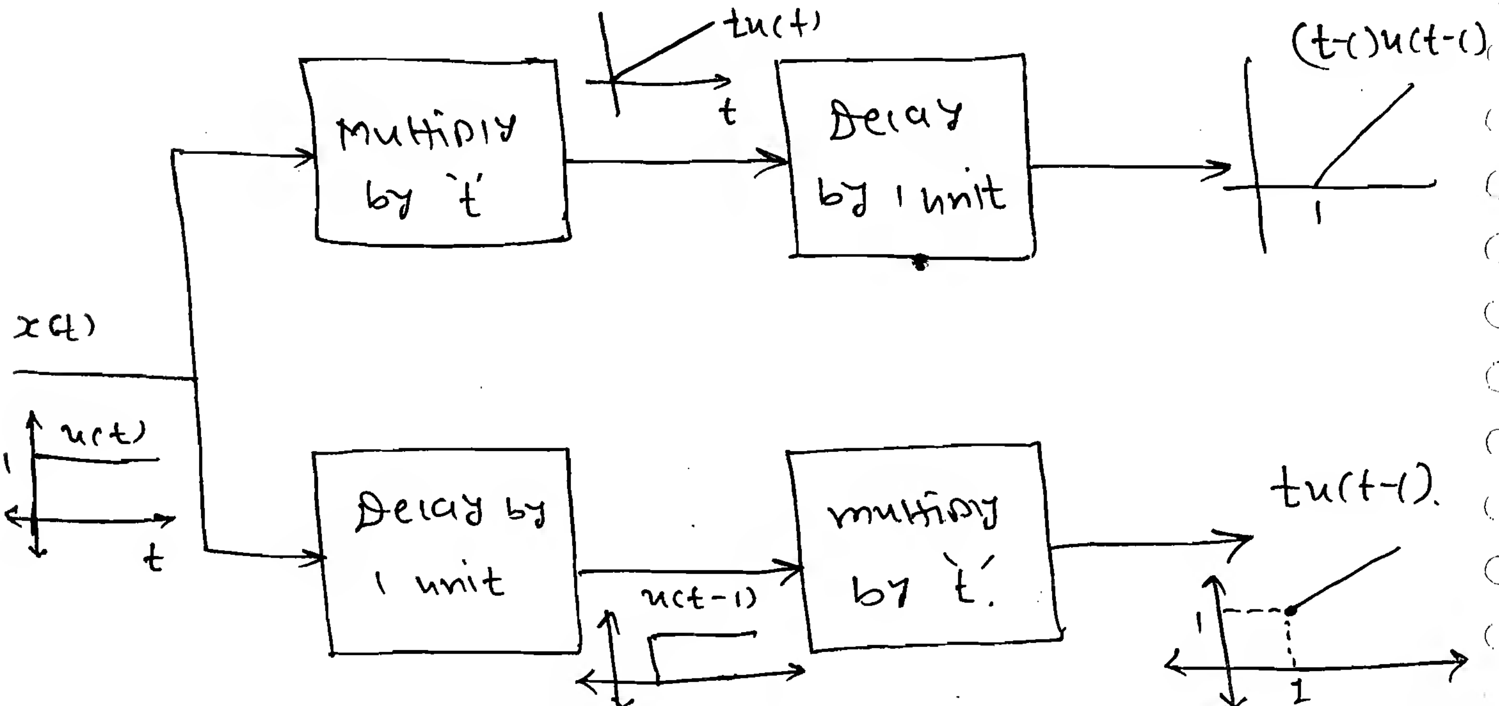
Sol^n: O/P due to delay input

$$y_1(t) = t\, x(t - t_0).$$

Delay O/P by 't_0' $\Rightarrow$ $t \Rightarrow t - t_0$.

$$\therefore\ y(t - t_0) = (t - t_0) \cdot x(t - t_0) \neq y_1(t)$$

So, Time Variant.



② $y(t) = e^{-x(t)}$.

Solⁿ: time Invariant

$y(t) = e^{-x(t-t_0)}$

$y(t-t_0) = e^{-x(t-t_0)}$

③ $y(t) = x^2(t)$.

Solⁿ: $y(t) = x^2(t-t_0)$

$y(t-t_0) = x^2(t-t_0)$

So, Time Invariant.

④ $y(t) = x(2t)$.

Solⁿ: I/P: $y(t) = x(2t-t_0)$.

O/P: $y(t-t_0) = x(2(t-t_0)) = x(2t-2t_0)$.

So, Time Variant.

⑤ $y(t) = x(-t)$.

Solⁿ:

I/P: $y(t) = x(-t-t_0)$.

O/P: $y(t-t_0) = x(-(t-t_0)) = x(-t+t_0)$.

So, Time Variant.

⇒ In General,

$$\boxed{x(\alpha t) = \text{T.V.} \quad \alpha \neq 1}$$

IMP

6 $y(t) = \frac{d\,x(t)}{dt}$.

Sol^n:

$$y(t-t_0) = \frac{d\,x(t-t_0)}{dt\,(t-t_0)} = \frac{d\,x(t-t_0)}{dt}$$

$\left(\because \frac{d}{dt}(t_0) = 0 \right.$ ↑ constant).

So, Time Invariant.

7 $y(t) = \begin{cases} x(t) & ; x(t) > 0 \\ 0 & ; x(t) < 0. \end{cases}$

Sol^n:

Amp. dependent → Non-linear.

Independent of time → T. I.

8 $y(t) = \begin{cases} x(t) & ; t > 0 \\ 0 & ; t < 0. \end{cases}$

Sol^n:

Amp. Independent ⇒ Linear

Time dependent ⇒ T. V.

* <u>discrete:</u>

① $y[n] = g[n].\, x[n]$.

Sol^n: $y[n]$ I/P : $g[n].\, x[n-n_0]$

O/P : $g[n-n_0].\, x[n-n_0]$

So, T. V.

② $y[n] = 3\, x[n]$.

Sol^n: I/P : $3\, x[n-n_0]$

O/P : $3.\, x[n-n_0]$ $\Big\} \rightarrow$ T. I.

$\frac{dy}{dx}$ $y \to D.V$
$x \to I.V$

(3) $\frac{dy(t)}{dt} + t \cdot y(t) = x^2(t).$

Sol^n: $t \cdot y(t) \downarrow$ T.V. $x^2(t) \downarrow$ N.L.

Linear
→ First degree of differentiation &
→ Co-efficient must be constant or I.V. (here I.V. is t)

(4) $\frac{d^2y(t)}{dt} + 4y(t) = 2x(t).$

Sol^n: $\frac{d^2y(t)}{dt} \downarrow$ L, $4y(t) \downarrow$ T.I.

So, Linear time Invariant (L.T.I.).

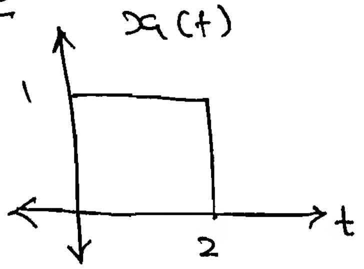
⇒ Because of co-efficient are fixed ⇒ T.I.

⇒ In any differential eq^n if all the co-efficient are fixed only with linear elements that is LTI system.

i) I/P O/P depending on time → V
ii) Time scaling. → V
iii) Modulator (signal multiply by sine or cos). → V

Page-(17)

P: 1.4.5.



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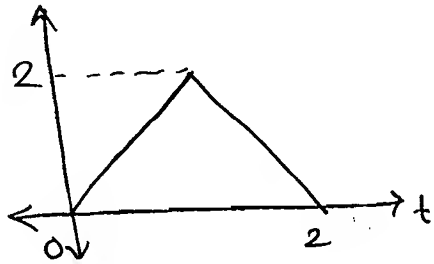
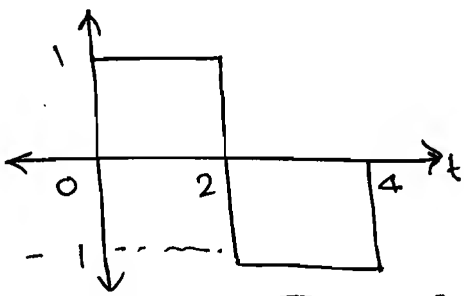
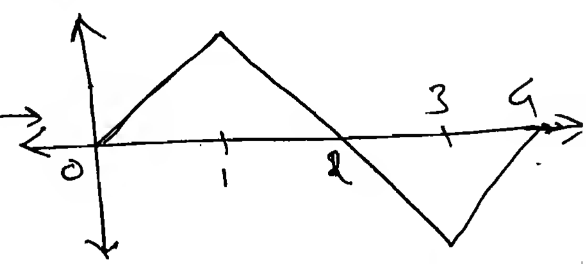


⇒



→

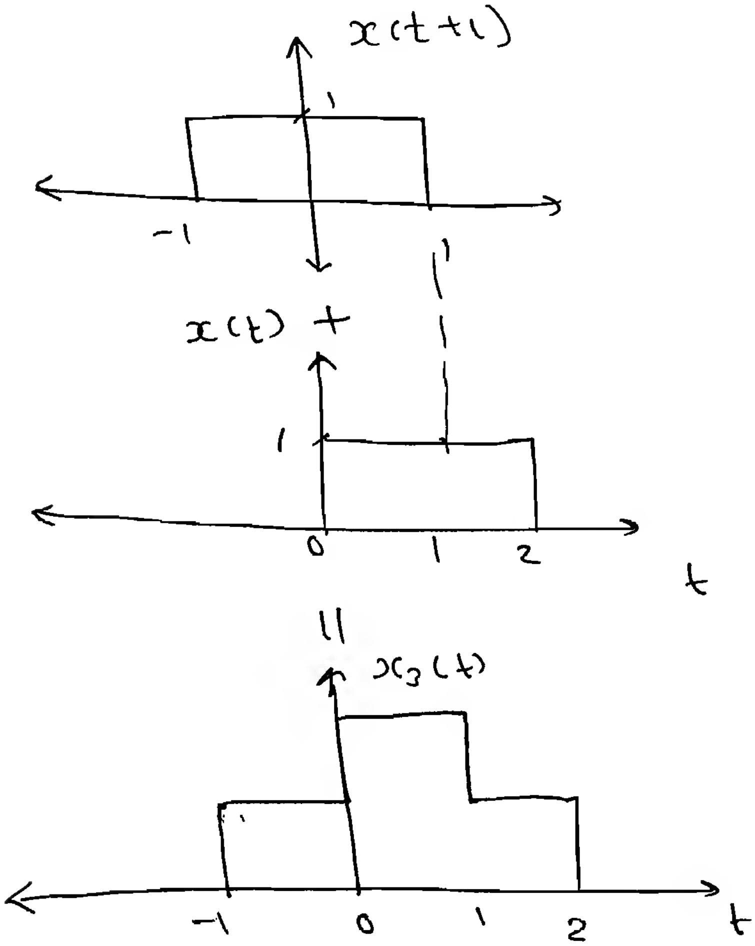


$\Rightarrow \quad x_2(t) = x_1(t) - x_1(t-2).$

$\big\downarrow$ LTI (given).

$\therefore \quad x_2(t) = y_1(t) - y_1(t-2).$

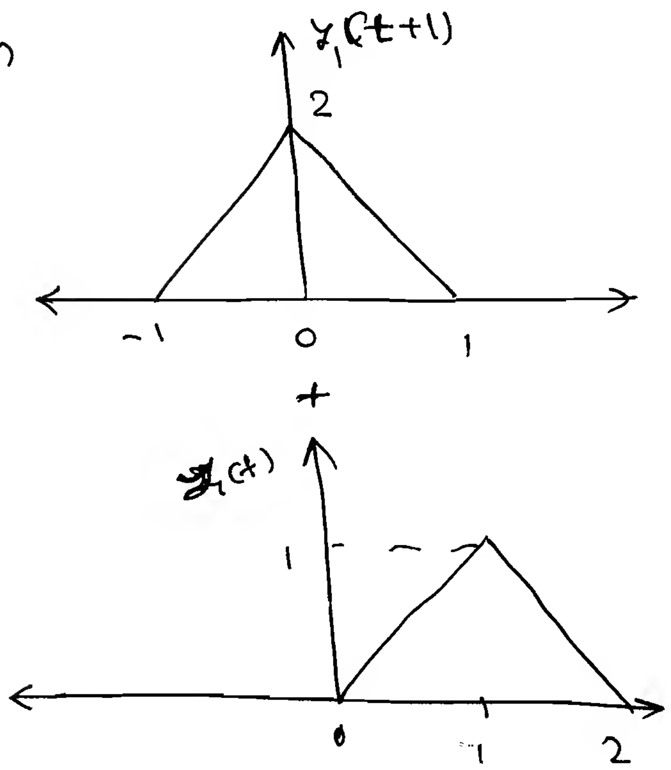
$\Rightarrow$



$x_3(t) = x_1(t+1) + x_1(t).$

So, $y_3(t) = y_1(t+1) + y_1(t).$

$\Rightarrow$



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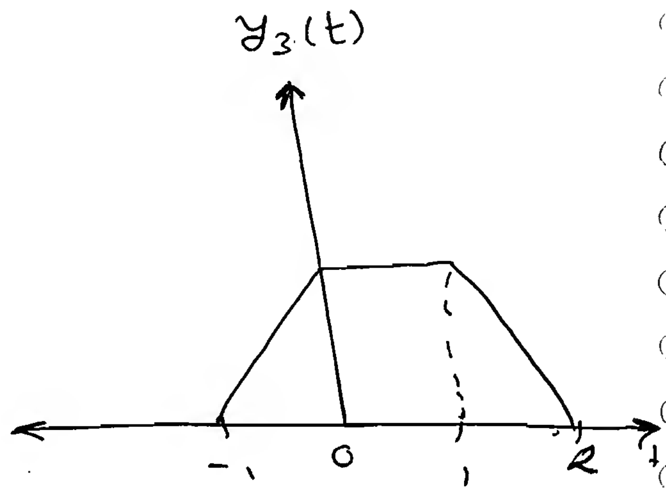


★ $x(t) \rightarrow y(t)$. ⇒ LTI.

Initial cond^n $y(0) = 0$.

⇒ For a LTI system same relation is valid for i/p and o/p.

[3] Causal and Non-Causal System.

⇒ A system is causal system if the present o/p depends only on the present i/p and past values of the i/p but not on future values i.e. Causal systems are non-anticipative.

e.g.: (i) $y(t) = (3t+1)\,x(t)$.

Causal.

(ii) $y(t) = x(t).\cos\omega_0 t$

causal.

$\boxed{Q}$

① $y(t) = \sin\{x(t)\}$.

Sol^n:

Causal.

② $y(t) = x\{\sin(t)\}$.

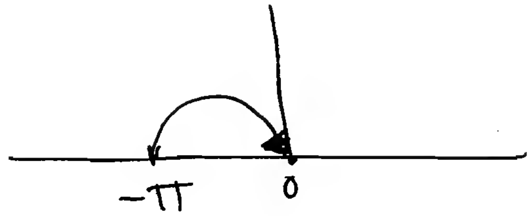
Sol^n:

$\sin t = 0$ for $t = \pm m\pi$, $m = 0, 1, 2, \ldots$

$\therefore y(-\pi) = x\{\sin(-\pi)\}$

$y(-\pi) = x(0)$.

$\Rightarrow \quad y(-\pi) = x(0).$



o/p is expecting future value.

So, Non-Causal.

(3) $$y(t) = \int_{-\infty}^{2t} x(\tau) \cdot d\tau.$$

Sol^n:

$$y(1) = \int_{-\infty}^{2} x(\tau) \cdot dt.$$

So, Non-Causal.

(4) $$y(t) = \int_{-\infty}^{t} x(\tau) \, d\tau.$$

Sol^n:

$$y(1) = \int_{-\infty}^{1} x(\tau) \cdot d\tau. \Rightarrow \text{Causal.}$$

(5) $$y[n] = 2x[n] + 3u[n+1].$$

Sol^n: Causal

$\hookrightarrow$ Not affecting causality.

(6) $$y[n] = \sum_{k=n_0}^{n} x[k]$$. "n_0" is finite.

Sol^n: Case - I: $n_0 > n$. $\Rightarrow$ Non Causal (NC).

$2 > 1$

$$y[1] = \sum_{k=2}^{1} x[k] = x[1] + x[2]$$

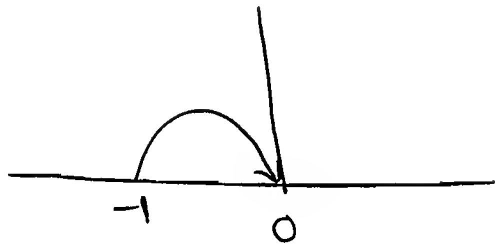
Case - II: $n_0 \leq n$.

$$y(2) = \sum_{k=1}^{2} x[k] = x[1] + x[2]. \Rightarrow \text{(c)}$$

$\Rightarrow$ So, Conditionally Causal.

$\Rightarrow$ for $n_0 = 0$.

$$\therefore \; y[n] = \sum_{k=0}^{n} x[k].$$



$$\therefore \; y[-1] = \sum_{k=0}^{-1} x[k] = x[-1] + x[0].$$

So, Non - Causal.

[7] $$y[n] = \sum_{k=-\infty}^{n} x[k].$$

Solⁿ: It is accumulator.

$$y[1] = \sum_{k=-\infty}^{1} x(k) = \cdots + x[-2] + x[-1] + x[0] + x[1].$$

So, Causal.

[8] $$y[n] = \frac{1}{2M+1} \sum_{k=-M}^{+M} x[n-k].$$

Solⁿ: It is Moving avg. system.

Let, $M = 1$

$$\therefore \; y[n] = \frac{1}{3} \sum_{k=-1}^{1} x[n-k].$$

$\therefore$

$\therefore\ y[n] = \frac{1}{3}\left[x[n+1] + x[n] + x[n-1] \right].$

So, Non - Causal system.

<u>Note:</u> Non - Causal systems can not be design when the independent variable is time. (t or n).

[9] $\dot{y}(t+4) + 2(t) = x(t+2).$

<u>Soln:</u> Put $t+4 = \tau$.

$\therefore\ \dot{y}(\tau) + 2(\tau-4) = x(\tau-2).$

So, Causal system.

$\Rightarrow$ Here, Present time is $t+4$.

[4] Static | Memory less &

Dynamic | with memory :

$\Rightarrow$ Static system is that in which o/p at perticular instant is depend only on input at ~~on~~ that instant only.

eg: ① $y(t) = e^{-(t+3)} \cdot x(t).$

Static.

$\boxed{Q}$ ① $y(t) = 2x(t) + 3$

Static

② $y[n] = g[n+3] \cdot x[n]$

Static.

③ $y[n] = x[3n]$.

Soln: $y[1] = x[3] \Rightarrow$ Dynamic.

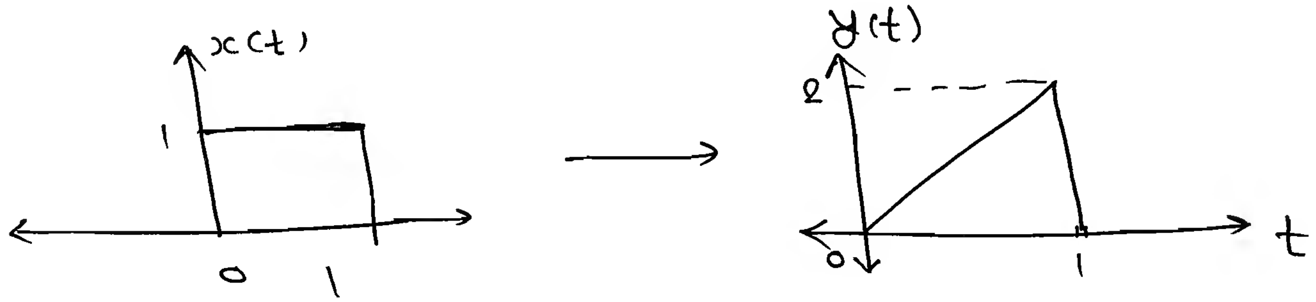
④ $y(t) = \frac{d}{dt}(x(t))$

Soln: differential terms is because of system is energy storing elements

So, System is alway dynamic.

$\Rightarrow$ All Static systems are causal system.

[a]



Soln: Before applying an input, O/P should not start $\Rightarrow$ Causal.

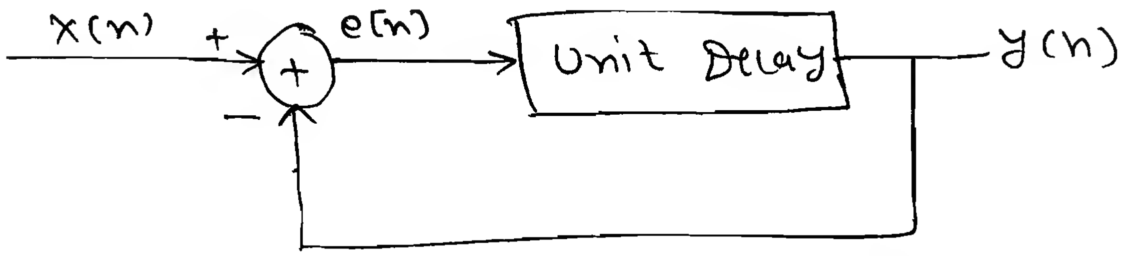
$\Rightarrow$ I/P is starting at '0' and O/P is also starting at '0'. So, causal system.

$$y(t) = 2\int_{-\infty}^{t} x(\tau)\,d\tau.$$

$\downarrow$

Causal & Dynamic.

a Consider the F/B system shown in figure, assume that $y[n] = 0$ for $n<0$.



⇒ Find $y(n)$ when the input is

(i) $x(n) = \delta[n]$.

(ii) $x(n) = u[n]$.

Solⁿ:

$$e[n] = x[n] - y[n].$$

$$\therefore \quad y[n] = e[n-1].$$

$$\therefore \quad y[n] = x[n-1] - y[n-1].$$

⇒ $n=0, \quad y[0] = x[-1] - y[-1] = 0 - 0 = 0.$

① $x[n] = \delta[n]$.

$\rightarrow y[0] = 0 - 0 = 0.$

$\therefore \quad y[1] = x[0] - y[0].$

$\longrightarrow y[1] = 1 - 0 = 1.$

$\rightarrow y[2] = x[1] - y[1].$

$= 0 - 1 = -1.$

$\therefore y[n] = [0, 1, -1, +1, -1, \dots].$

(OR) Using z-transform.

$\Rightarrow \dfrac{z^{-1}}{1+z^{-1}} \Rightarrow$ IZT $\Rightarrow (-1)^{n-1} \cdot u(n-1)$.

② $x[n] = u[n]$.

O/P $y[n] = \{0, 1, 0, 1, 0, \ldots\}$.

Note:

$\Rightarrow$ Present o/p depends only on present i/p and past input then it is Finite Impulse Response [FIR].

$\Rightarrow$ If it also depends on past o/p it is IIR (Infinite impulse Response) filter. FB is there $\Rightarrow$ Recursive.

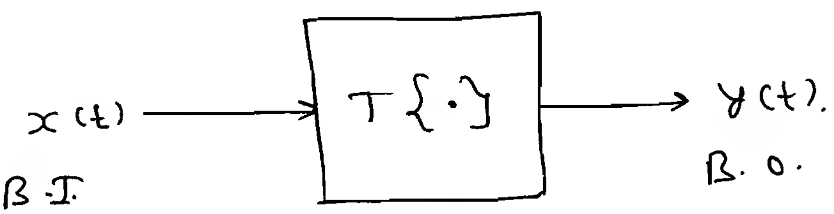
[5] Stable & Unstable Systems:

$\Rightarrow$ It is a magnitude concept.

$\Rightarrow$ Boundedness $\Rightarrow$ Bounded amplitude.

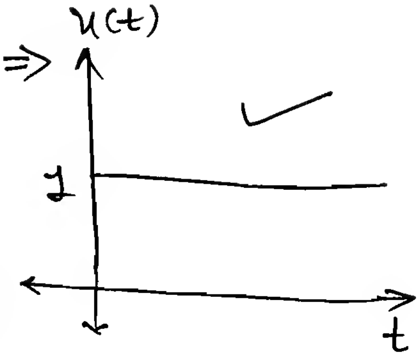
$\Rightarrow$ B.I.B.O.

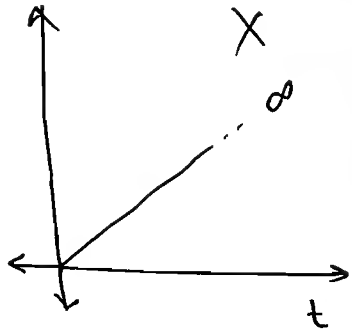
$\Rightarrow$

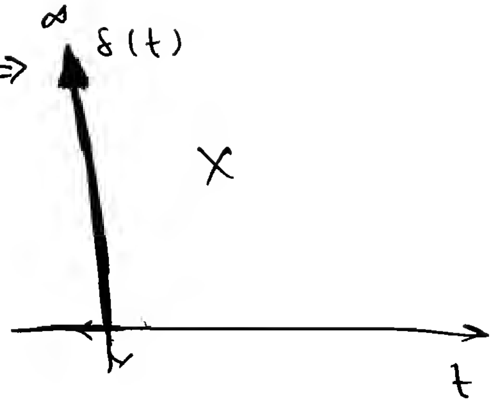


If $|x(t)| \leq M_x < \infty$ then
$|y(t)| \leq M_y < \infty$. $\rightarrow$ Stable.

$\Rightarrow$ For finite input, o/p should be finite.

$\Rightarrow$ 

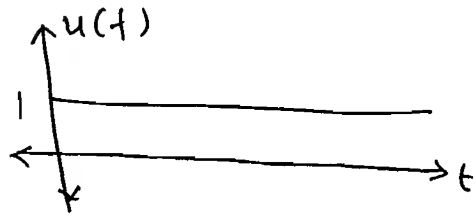
$\Rightarrow$ 

$\Rightarrow$ 

$\Rightarrow$ Sine & Cos $\Rightarrow$ ✓ stable.

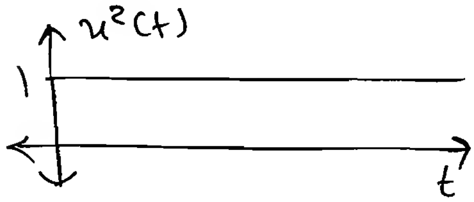
$\boxed{\text{Q}}$

① $y(t) = x^2(t)$.



Solⁿ: If $x(t) = u(t)$

$\therefore y(t) = u^2(t) = u(t)$.



Stable

* Alternative

$\rightarrow$ Magnitude Concept.

$\rightarrow y(t) = |x(t)|^2$

$= (\text{finite})^2 = \text{finite} \Rightarrow$ Stable.

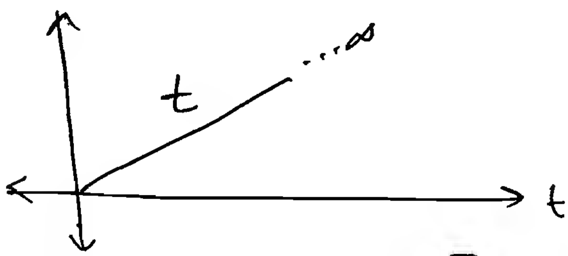
② $y(t) = x(2t)$.

Solⁿ: Stable.

③ $y(t) = t \cdot x(t)$.

Solⁿ: if $x(t) = u(t)$

$\Rightarrow y(t) = t \cdot u(t)$



So, Unstable.

$\boxed{Q}$ (4) $y(t) = \frac{d}{dt} . x(t).$

Solⁿ:

let, $x(t) = u(t).$

$y(t) = \frac{d}{dt} \cdot u(t) = \delta(t).$ (Unbounded).

So, Unstable.

(5) $y(t) = x(t). \cos \omega_c t.$

Solⁿ: $|y(t)| = |x(t). \cos \omega_c t|$

$= |x(t)| \cdot \cos \omega_c t \rightarrow$ max value +1 & -1.

$=$ finite $=$ Stable.

(6) $y(t) = \int_{-\infty}^{t} |x(\tau). \cos \omega_c t| \, dt.$

Solⁿ: $y(t) = \int_{-\infty}^{t} (\text{finite}) \, . dt.$

$= \infty$

$\Rightarrow$ Unstable.

(7) $y[n] = 2x[n] + 3.$

Solⁿ: Stable finite + finite = finite.

$\Rightarrow$ for discrete we have $\delta[n], u[n].$

(8) $y[n] = e^{x[n]}.$

Solⁿ: $|y[n]| = e^{|x[n]|} =$ finite $\Rightarrow$ Stable.

⑨ $y[n] = \sum_{n=n_0}^{\infty} x[k]$ "n_0 is finite".

Sol^n:

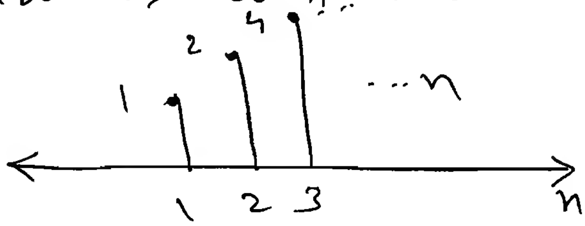
Let, $x[k] = 1$.

$\therefore y[n] = \sum_{n=n_0}^{n} (1)$.

$= 1 + 1 + 1 + \cdots$ n terms.

$= n$.

as $n \to \infty \Rightarrow$ amp. $\to \infty \Rightarrow$ Unstable



⑩ $y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$.

Sol^n:

$x[k] = 1$,

$y[n] = \sum_{k=n-n_0}^{n+n_0} (1)$.

Put, $k - n + n_0 = m$.

$\therefore y[n] = \sum_{m=0}^{m=2n_0} (1)$.

$\therefore y[n] = (2n_0 + 1)$

$\Downarrow$

Stable.

[6] Invertible & Inverse System :-

⇒ A System is an Invertible if different inputs (different amplitudes) leads to different outputs i.e. two different i/ps for a given System should not produce Same o/p.

$$x(t) \longrightarrow \boxed{T\{\cdot\}} \xrightarrow{y(t)} \boxed{T^{-1}\{\cdot\}} \longrightarrow x(t)$$

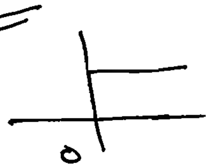
$$\underline{\underline{T \cdot T^{-1} = 1}}$$

e.gs:

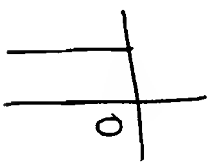
① $y(t) = x^2(t)$.

Sol^n:

i/p $\longrightarrow$ o/p



$u(t) \longrightarrow u(t)$.



$-u(t) \longrightarrow u(t)$.

So, Non- Invertible.

② $y(t) = x(t-4)$.

Sol^n:

$\delta(t) \longrightarrow \delta(t-4)$.

$-\delta(t) \longrightarrow -\delta(t-4)$

$u(t) \longrightarrow +u(t-4)$.

$-u(t) \longrightarrow -u(t-4)$.

So, Invertible & Inverse is $y(t+4)$.

③ $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau.$

Soln:

$i/p \longrightarrow o/p.$

$\delta(t) \longrightarrow u(t).$

$-\delta(t) \longrightarrow -u(t).$

$u(t) \longrightarrow t\,u(t).$

$-u(t) \longrightarrow -t\,u(t).$

So, Invertible & Inverse is $\frac{dy(t)}{dt}$.

*

③ $y(t) = \frac{d}{dt}(x(t)).$

Soln:

$i/p \longrightarrow o/p$

$2 \longrightarrow 0$

$3 \longrightarrow 0$

$4 \longrightarrow 0$

So, Non-Invertible.

④ $y[n] = n \cdot x[n].$

Soln:

$i/p \longrightarrow o/p.$

$\delta[n] \longrightarrow n \cdot \delta[n]$

$= 0 \cdot \delta[n] = 0$ $(\because n_0 = 0)$. Property of $\delta[n]$).

$-\delta[n] \longrightarrow -n \cdot \delta[n]$

$= 0 \cdot \delta[n] = 0.$

So, Non- Invertible.

⑤ $y[n] = x[n] \cdot x[n-3].$

Soln:

$i/p \longrightarrow o/p$

$\delta[n] \longrightarrow \delta[n] \cdot \delta[n-3] = 0$

$-\delta[n] \longrightarrow (-\delta[n]) \cdot (-\delta[n-3]) = 0$

So, Non- Invertebile.

[6] $y[n] = \sum_{k=-\infty}^{n} x[k].$

Sol^n: Cont. Discrete

$\Sigma \Rightarrow \int$

i/p $\longrightarrow$ o/p

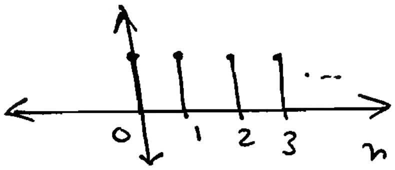
$\delta[n] \longrightarrow u[n].$

$-\delta[n] \longrightarrow -u[n].$

So, Invertible.

$\Rightarrow$ i/p

$u[n]$



$\longrightarrow \sum_{k=0}^{n} (1) = (n+1)u[n].$

So, Invertible.

Inverse is $y[n] - y[n-1]$.

$$= \sum_{k=-\infty}^{n} x[k] - \sum_{k=-\infty}^{n-1} x[k].$$

$$= x[n] + \cancel{\sum_{k=-\infty}^{n-1} x[k]} - \cancel{\sum_{k=-\infty}^{n-1} x[k]}$$

$$= x[n].$$

(7) $y[n] = x[n] . \sin\left[\frac{5\pi n}{6}\right].$

Sol^n: i/p $\longrightarrow$ o/p

$\delta[n] \longrightarrow \delta[n] . \sin\left[\frac{5\pi n}{6}\right].$

$= \sin\left(\frac{5\pi \cdot 0}{6}\right) . \delta[n] \quad (\because n_0 = 0).$

$= 0.$

$\rightarrow \quad -\delta[n] \longrightarrow -\delta[n] \cdot \sin\left(\frac{5\pi n}{6}\right)$

$= 0.$

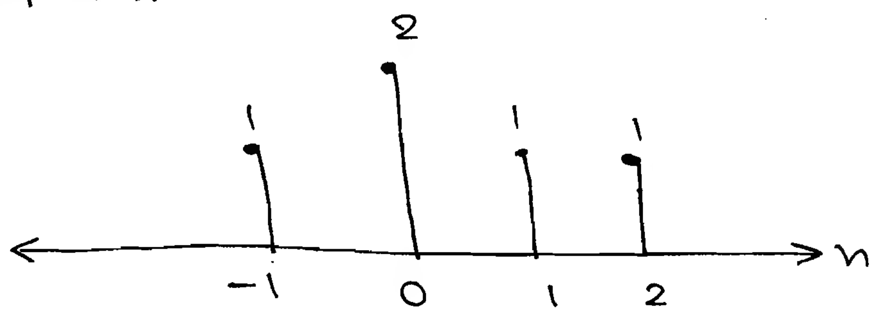
So, Non-Invertible.

☆ LTI Systems :-

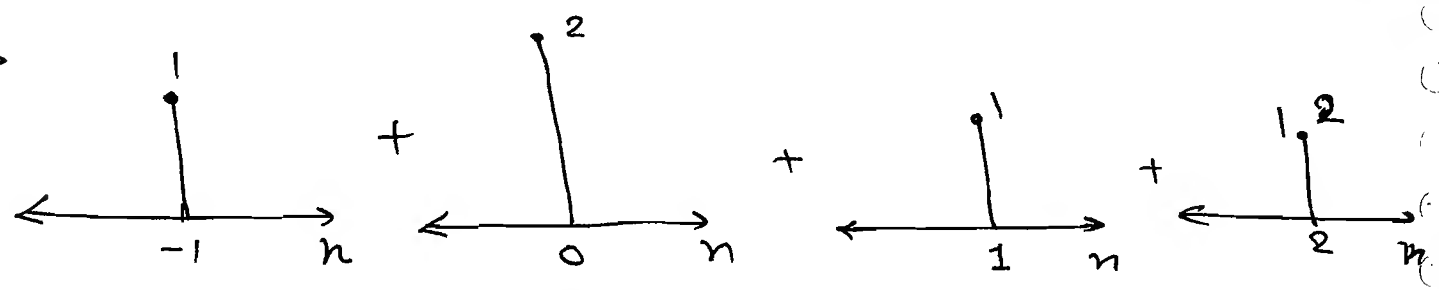
⇒ A LTI Systems is represented with respect to impulse response. (if input is impulse, output is impulse response).

⇒ Sifting property states that any signal can be produced as Combination of impulses.

e.g.



⇒

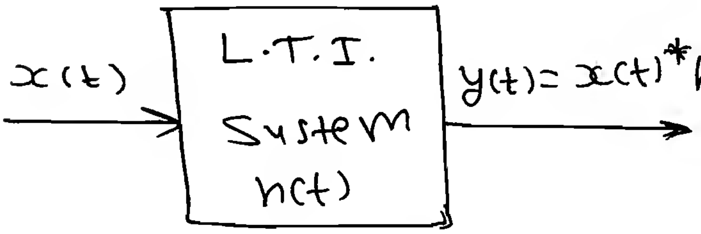


$x[-1]\,\delta[n+1]$ + $x[0]\,\delta[n]$ + $x[1]\,\delta[n-1]$ + $x[2]\cdot\delta[n-2]$

⇒ Convolution is formal mathmatical operation, just as multiplication, addition and integration. Addition takes two numbers and produces a third number,

while convolution takes two signals and produces a third signal.

* Continuous Convolution



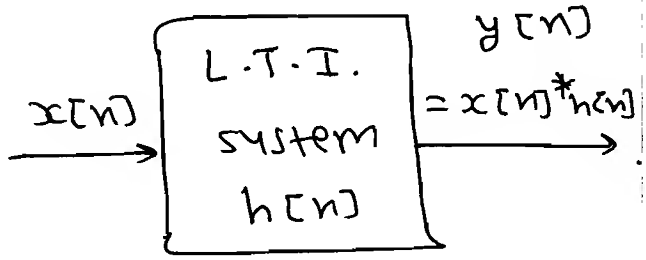
$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\cdot h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} x(t-\tau)\cdot h(\tau)\, d\tau$$

Steps:

1. $x(t) \longrightarrow x(\tau)$,
   $h(t) \longrightarrow h(\tau)$.
2. Folding — $x(-\tau)$, $h(-\tau)$.
3. Shifting — $x(t-\tau)$, $h(t-\tau)$
4. multiplication — $x(t-\tau)\cdot h(\tau)$, $h(t-\tau)\cdot x(\tau)$.
5. Integration.

Discrete Convolution



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\cdot h[n-k].$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[n-k]\cdot h[k].$$

Steps:

1. $x[n] \longrightarrow x[k]$,
   $h[n] \longrightarrow h[k]$.
2. Folding — $x[-k]$, $h[-k]$
3. Shifting — $x[n-k]$, $h[n-k]$.
4. Multiplation — $x[k]\cdot h[n-k]$, $x[n-k]\cdot h[k]$
5. Summation.

→ By using Convolution we are finding the zero state response for a given input and system. (Zero initial Condition).

→ Sliding one signal over the folded and shifted version of the other signal is a concept of Convolution.

$$P_1(s) = s^2 + 2s + 1$$

$$P_2(s) = s^2 + 3s + 4.$$

$s^2 + 2s + 1$ (fixed)

$4 + 3s + s^2$ (folding) ~~3s+~~

$4 + 3s \quad s^2 \longrightarrow = s^4$

$4 \quad 3s + s^2 \longrightarrow = 3s^3 + 2s^3 = 5s^3$

$4 + 3s + s^2 \longrightarrow = 4s^2 + 6s^2 + s^2 = 11s^2$

$4 + 3s \longrightarrow = 8s + 3s = 11s$

$4 \longrightarrow = 4$

$$s^4 + 5s^3 + 11s^2 + 11s + 4.$$

*

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

→ Shifting

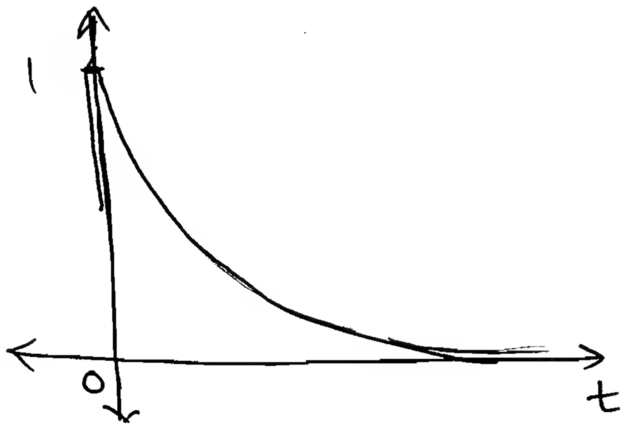
① folding

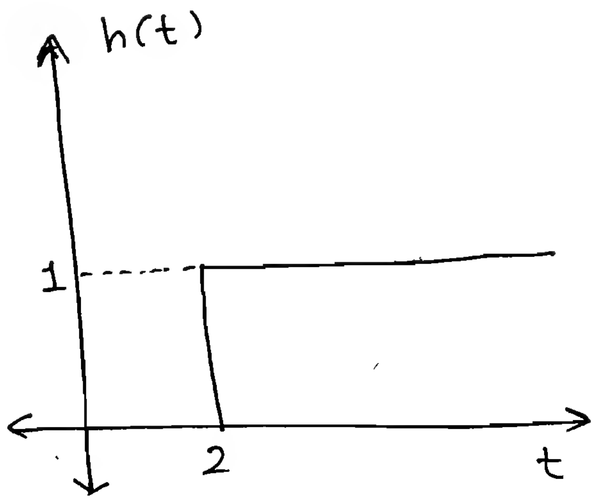
③ Multipilication

④ Integration.

P-1 Find the convolution of the two signal $e^{-3t} \cdot u(t)$ & $h(t) = u(t-2)$.

Soln: $x(t) = e^{-3t} \cdot u(t)$.





Steps:

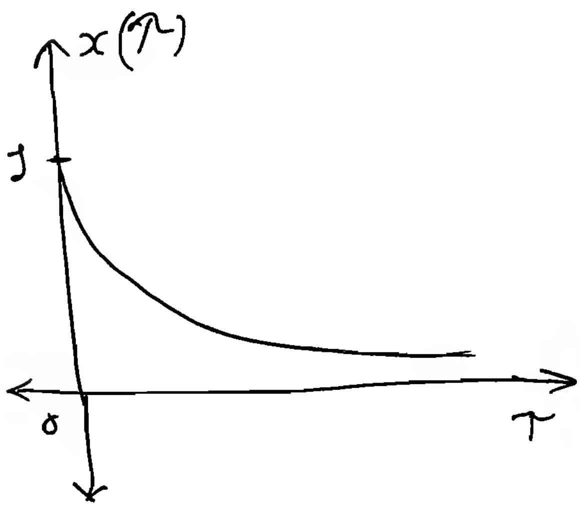
① Obtaining the limits of $y(t)$:-

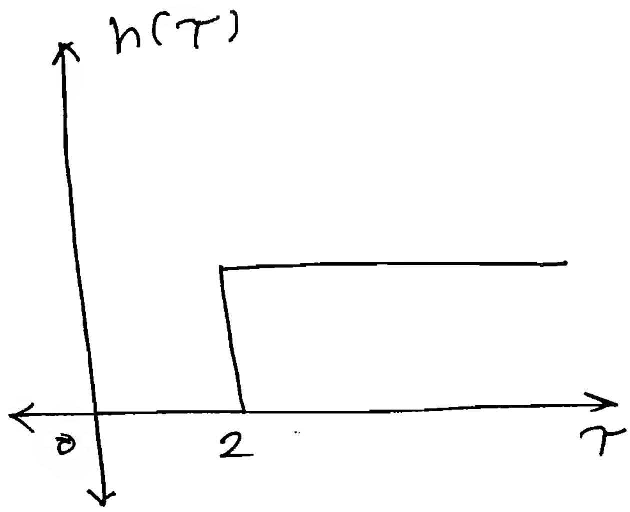
Sum of lower limits $< t <$ sum of upper limits.

$\therefore \quad 0 + 2 < t < \infty + \infty$

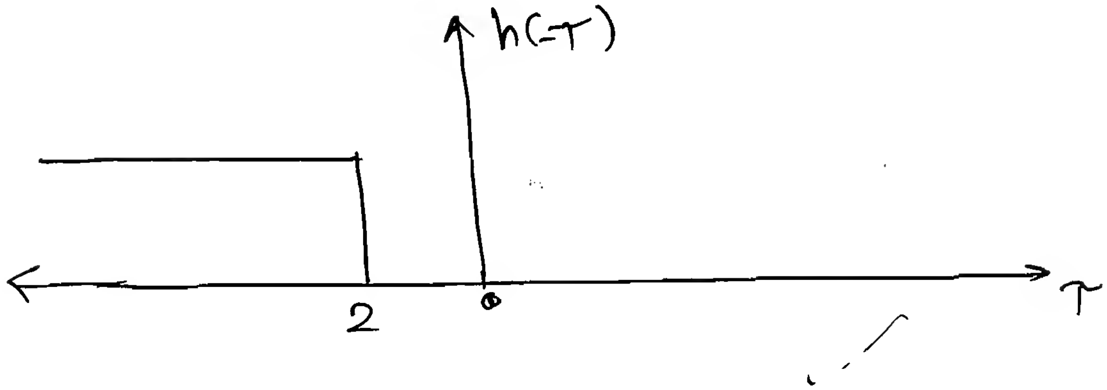
$\Rightarrow \quad \boxed{2 < t < \infty}$

② Change axis from "t" to "τ".

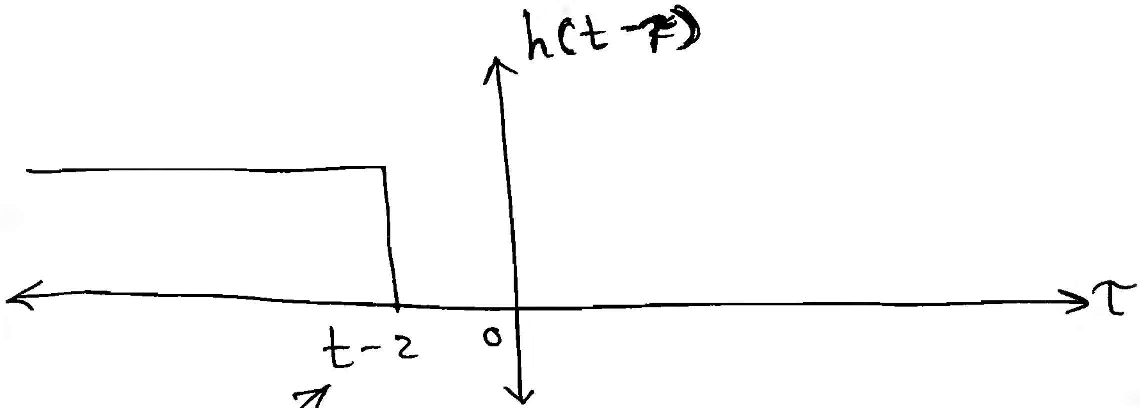




③ Folding / Flipping:



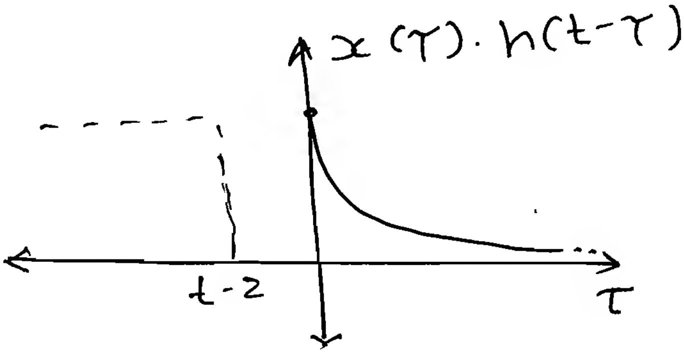
④ Shifting



Add variable i.e here (-2) to the t.

⑤ Multiplication:

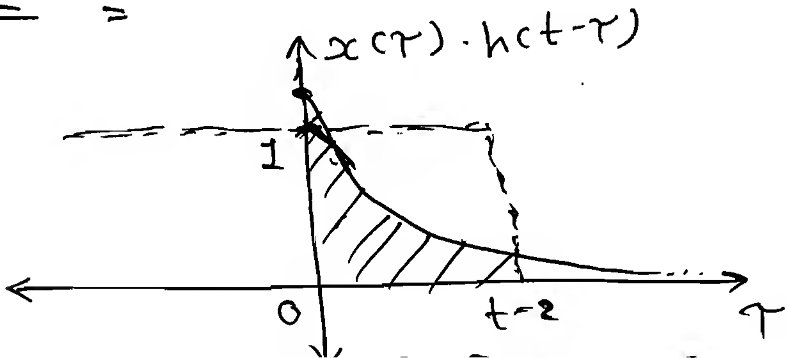
Case-(i) $t-2<0$.



$y(t)=0$ for $t<2$

as. No overlapping

Case-(ii) $t-2>0$



$$\Rightarrow \quad y(t) = \int_0^{t-2} e^{-3\tau} \cdot u(\tau) \cdot d\tau .$$

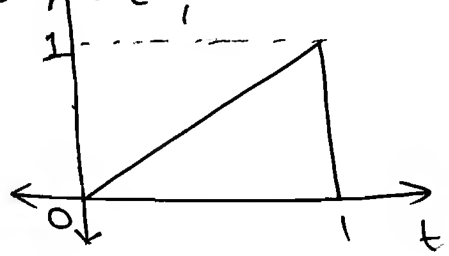
$$\therefore \boxed{y(t) = \frac{1 - e^{-3(t-2)}}{3} \quad ; \quad t > 2.}$$

Note:

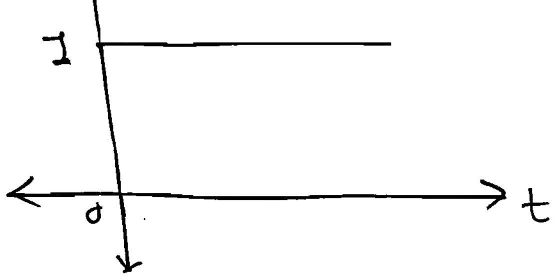
→ Convolution of two causal system is causal.

$\boxed{Q-2}$ Find the convolution of the signals shown in figure?

$x(t) = t \,, \; 0 < t < 1$



$h(t) = 1 \,, \quad t > 0.$

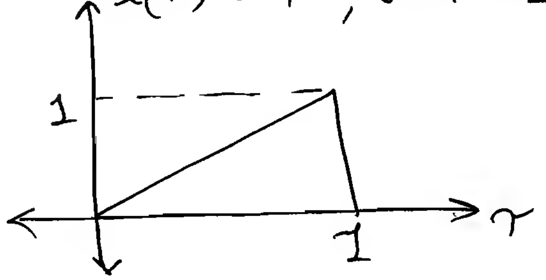


Sol^n:

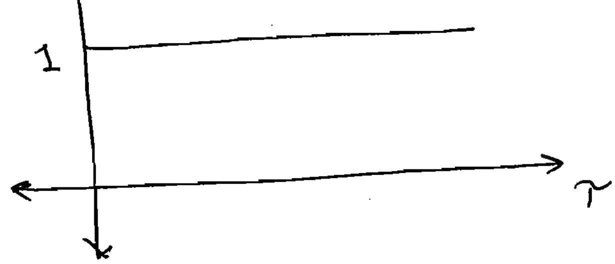
(1) $\quad 0 + 0 < t < 1 + \infty .$

$$\Rightarrow \quad \boxed{0 < t < \infty} \, .$$

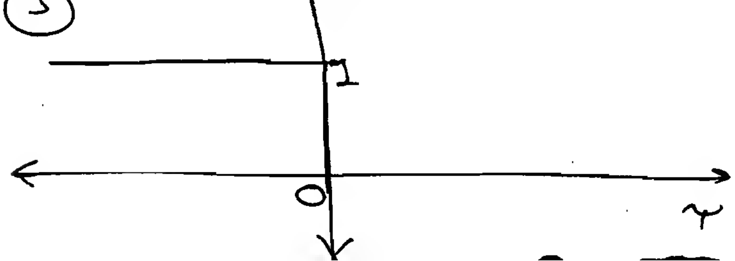
(2) $\quad x(\tau) = \tau \,, \; 0 < \tau < 1.$



$h(\tau) = 1 \,, \quad \tau > 0$

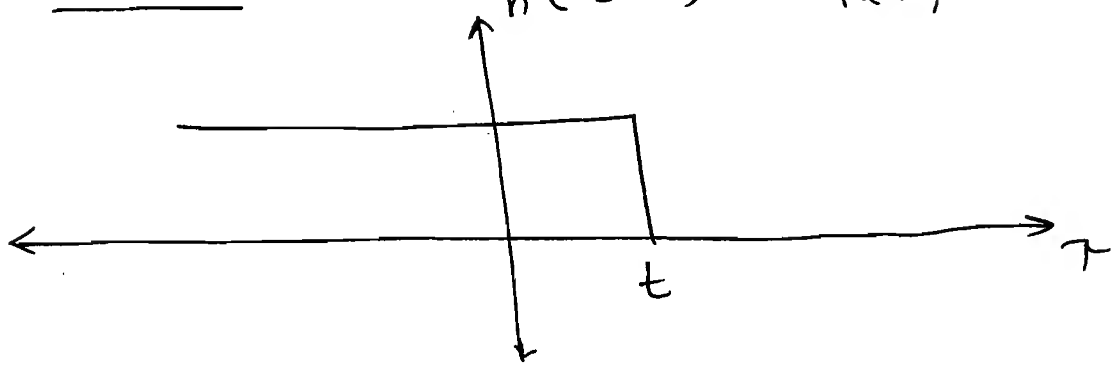


(3) $\quad h(-\tau)$



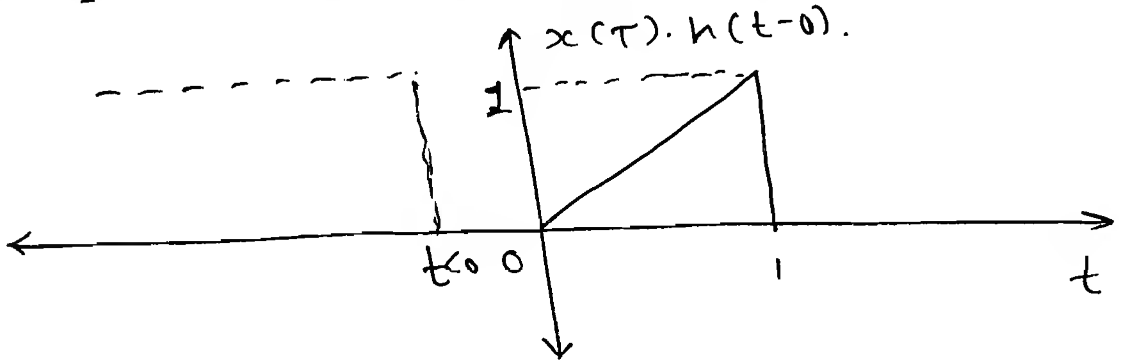
④ Shifting

$h(t-0)$ Here, Variable is 0.

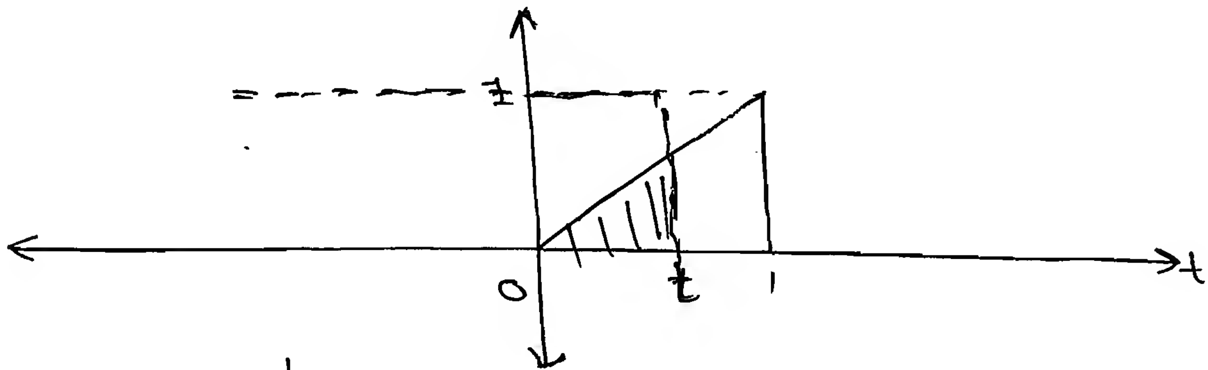


⑤ Multiplication & Integration.

Case-(i) $t<0 \Rightarrow y(t)=0$



case- (ii) $0<t<1$



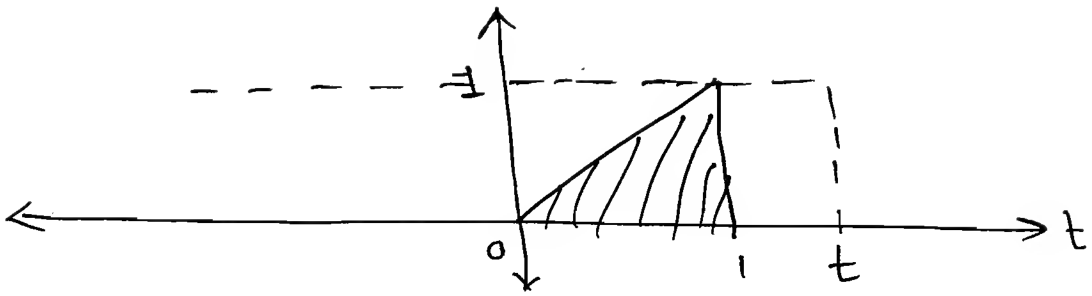
$$y(t) = \int_0^t (\tau) \cdot (1) \cdot d\tau.$$

$$y(t) = t^2/2 \; ; \quad 0<t<1.$$

Case-(iii) $t>1$

$$y(t) = \int_0^1 (\tau).\ (1)\ d\tau = \left[\frac{\tau^2}{2}\right]_0^1 = 1/2 \; ; \; t>1$$

⇒

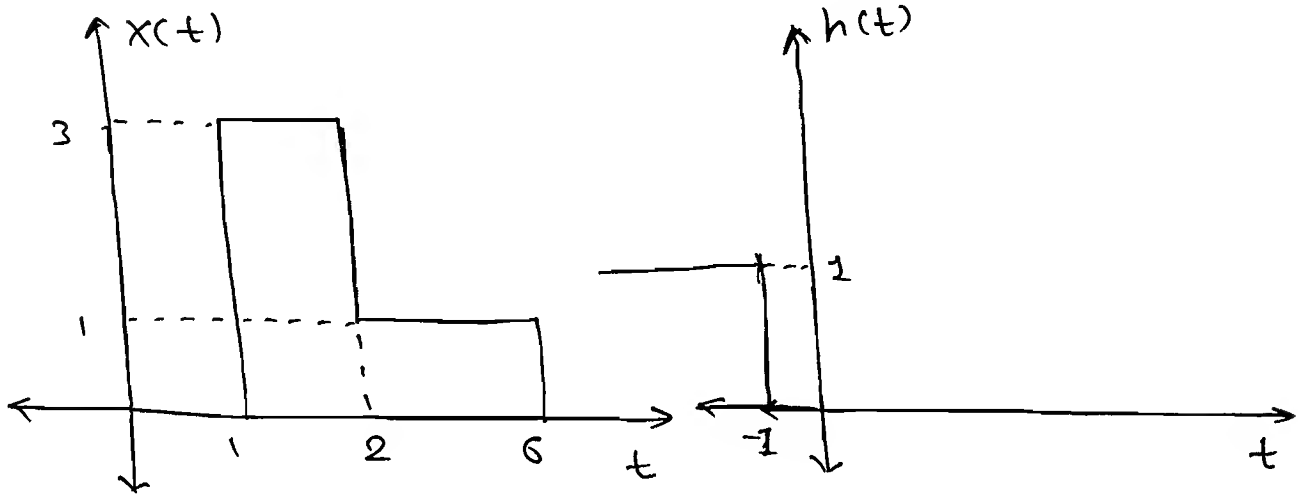


So, $y(t) = 0 \; ; \; t < 0$

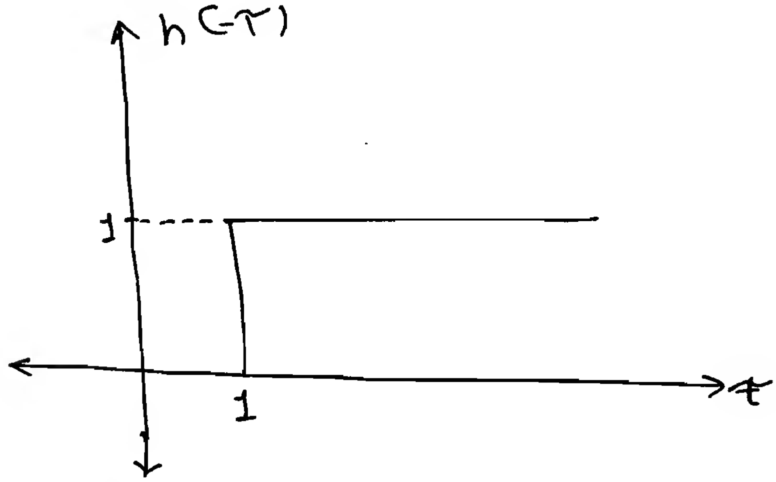
$= t^2/2 \; ; \; 0 < t < 1$

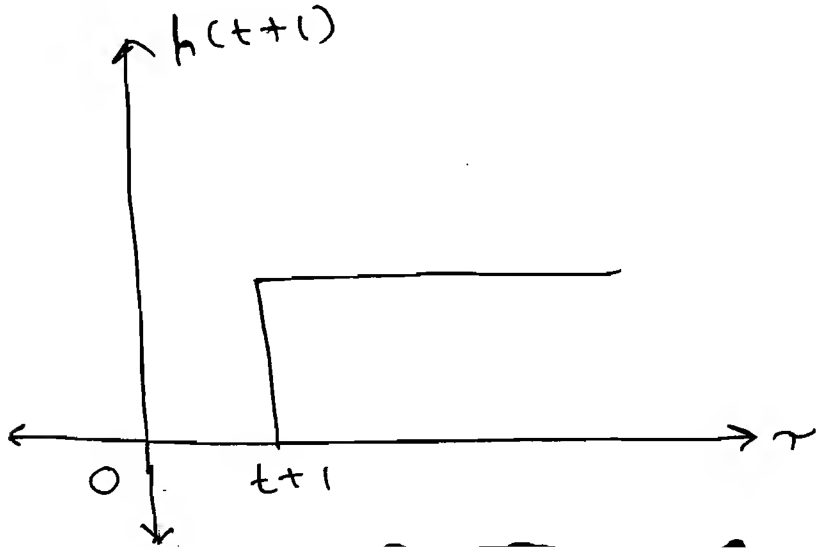
$= 1/2 \; ; \; t > 1.$

$\boxed{P-3}$

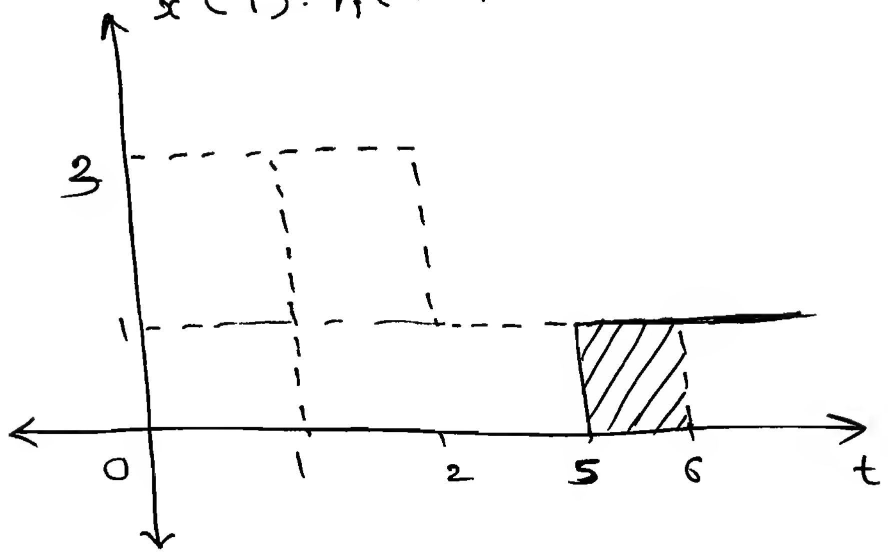


Soln:



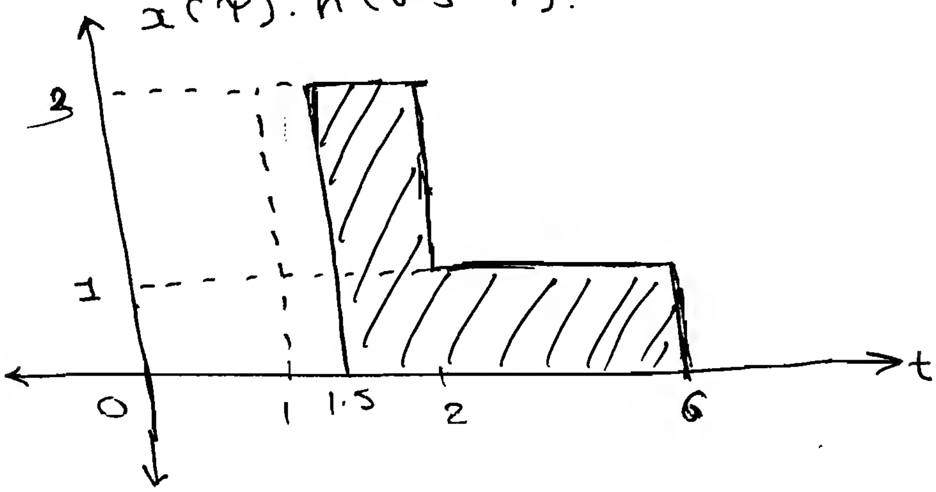


$\Rightarrow$ at $\underline{t=4}$.



$$y(t) = \int_{5}^{6} (1)(1).\, d\tau = 1.$$

$\Rightarrow$ at $t = 0.5$



$$y(t) = \int_{1.5}^{6} x(\tau).\, t(0.5-\tau).\, d\tau.$$

$$= \int_{1.5}^{2} (3).\, d\tau + \int_{2}^{6} (1).\, d\tau.$$

$$= 3(2-1.5) + (6-\cancel{1.5}\,2)$$

$$= 1.5 + 4$$

$$y(t) = 5.5.$$

P 2.1.4. Suppose $z(t) = \int_{-\infty}^{+\infty} x(-\tau + a)\, h(t+\tau)\, d\tau$.

Express $z(t)$ in terms of $y(t) = x(t) * h(t)$.

Soln:

take $t + \tau = \lambda$.

$\Rightarrow t = \lambda - \tau$. $\quad dt = d\lambda$

$-\tau = t - \lambda$.

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(t-\tau).h((\tau).d\tau.$$

$$= \int_{-\infty}^{\infty} x(\tau).h(t-\tau).d\tau.$$

$$z(t) = \int_{-\infty}^{+\infty} x(t-\lambda+a).\, h(\lambda)\, d\lambda.$$

$$= \int_{-\infty}^{\infty} x(\underline{(t+a)} - \lambda).\, h(\lambda)\, d\lambda.$$

$$\therefore \boxed{z(t) = x(t+a) * h(t+a)}$$

P 2.1.5.

(a) $x(t+5) * \delta(t-7) =$ ______.

(b) $x(t) * \delta(at+b) =$ ______.

Soln: (a) $x(t+5) * \delta(t-7)$

$= x(t+5-7)$ $\quad$ ($\because x(t) * \delta(t-t_0) = x(t-t_0)$.)

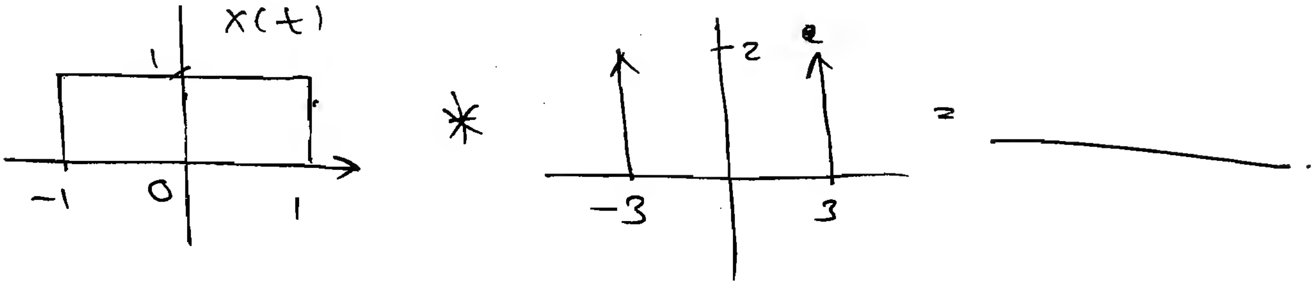
$= x(t-2)$.

(b) $x(t) * \delta(at + b)$

$$= x(t) * \left[\frac{1}{a}\,\delta(t + b/a)\right].$$

$$= \frac{1}{a}\,x(t + b/a).$$

(c)



Soln.
$$x(t) = u(t+1) - u(t-1).$$
$$h(t) = \delta(t+3) + \delta(t-3).$$

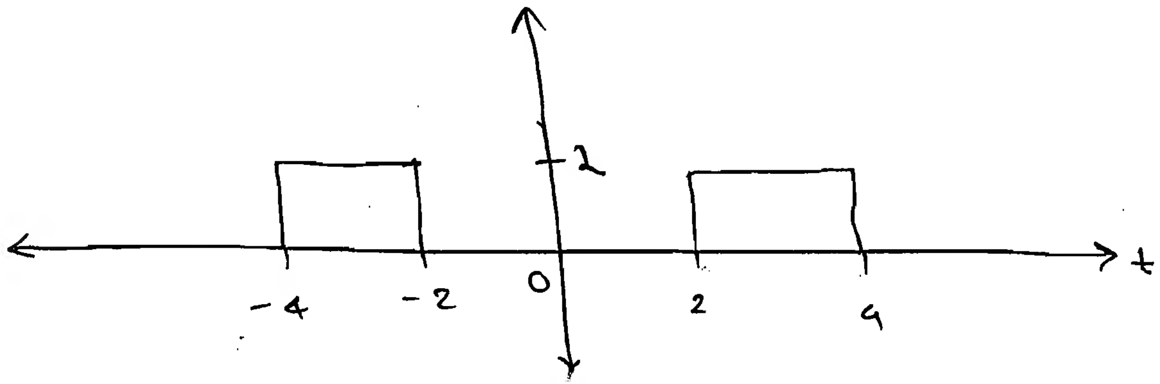
$$\therefore\ y(t) = x(t) * h(t)$$

$$= [u(t+1) - u(t-1)] * [2\delta(t+3) + 2\delta(t-3)].$$

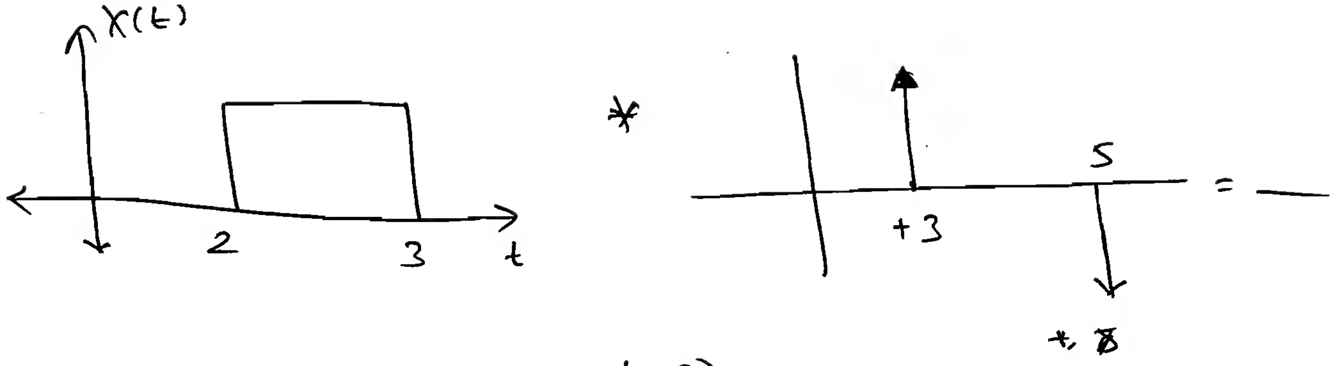
$$= 2[u(t+1) * \delta(t+3) + u(t+1) * \delta(t-3).$$
$$- u(t-1) * \delta(t+3) - u(t-1) * \delta(t-3)]$$

$$= 2[u(t+1+3) + u(t+1-3) - u(t-1+3)$$
$$- u(t-1-3)]$$

$$= 2[u(t+4) + u(t-2) - u(t+2)$$
$$- u(t-4)]$$



a



Soln:

$x(t) = u(t-2) - u(t-3).$

$h(t) = \delta(t-3) - \delta(t-5).$

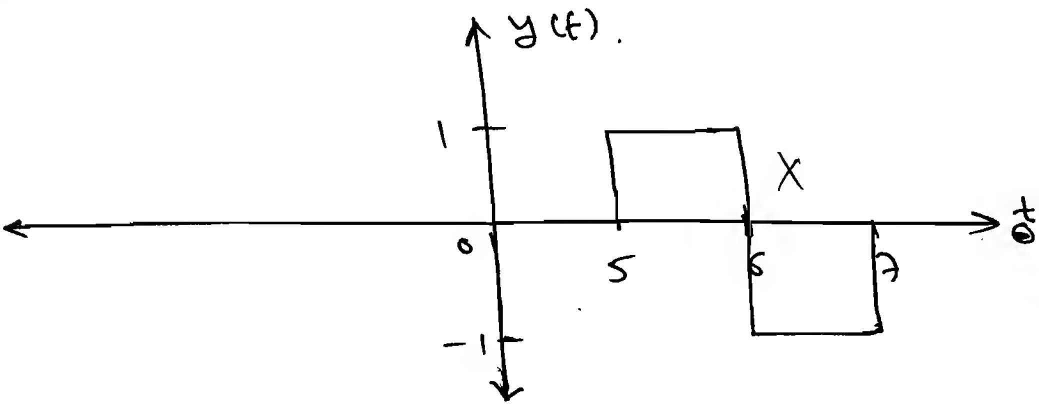
$\therefore\ y(t) = x(t) * h(t).$

$= u(t-2) * \delta(t-3) - u(t-3) * \delta(t-3)$

$+ u(t-2) * \delta(t-5) - u(t-3) * \delta(t-5).$

$= u(t-5) - u(t-6) + u(t-7) - u(t-8).$

$\therefore\ y(t) = u(t-5) - 2u(t-6) \mp u(t-7) + u(t-8)$



* Convolution property:

$$\boxed{x(t) * \delta(t-t_0) = x(t-t_0).}$$

e.g. ① $x(t) * \delta(t-13)$

$= x(t-13).$

② $x(t+5) * \delta(t-13).$

$= x(t+5-13)$

$= x(t-8).$

③ $x(t) * \delta(2t+3)$.

$$= x(t) * \left[\frac{1}{2}\,\delta(t+3/2)\right] \qquad \left(\because\ \delta(\alpha t) = \frac{1}{|\alpha|}\,\delta(t)\right).$$

$$= \frac{1}{2}\cdot\left[x(t) * \delta(t+3/2)\right]$$

$$= \frac{1}{2} \times x(t+3/2).$$

$\boxed{\text{P 2.1.3. (b)}}$

The impulse response of the Continuous time system is given by $h(t) = \delta(t-1) + \delta(t-3)$. The value of the step response at $t=2$ is.

(a) 0 (B) 1 (C) 2 (D) 3.

Soln:

$x(t) = u(t)$.

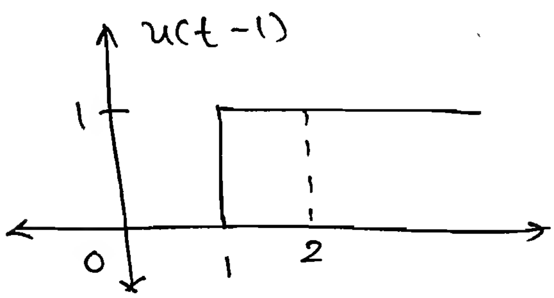
$y(t) = h(t)$.

$$y(t) = x(t) * h(t).$$

$$\therefore\ y(t) = u(t) * [\delta(t-1) + \delta(t-3)].$$

$$y(t) = u(t-1) + u(t-3).$$

$$\therefore\ y(2) = u(1) + u(-1) = 1 + 0 = \cancel{2}\ 1.$$



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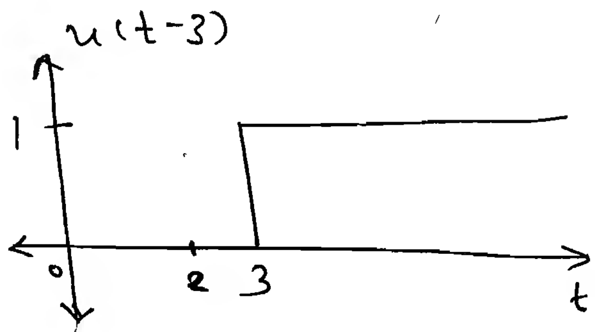


So, Ans ⇒ (B) 1.

$\boxed{P\ 2.1.6}$ Explain the difference between the following operations?

(a) $[e^{-t}u(t)]\,\delta(t-1)$.

$\Rightarrow$ it is product property of impulse

i.e. $\boxed{\begin{aligned} & x(t)\cdot\delta(t-t_0) \\ &= x(t-t_0)\cdot\delta(t-t_0). \end{aligned}}$

So, $[e^{-t}\cdot u(t)]\,\delta(t-1)$

$= e^{-1}\cdot u(1)\cdot\delta[t-1]$ $\quad(\because t_0=1)$.

$= e^{-1}\cdot\delta[t-1]$.

(b) $\int_{-\infty}^{+\infty} e^{-t}\cdot u(t)\cdot\delta(t-1)\,dt$.

$\Rightarrow$ It is a sifting property of impulse.

i.e. $\boxed{\int_{t_1}^{t_2} x(t)\cdot\delta(t-t_0)\,dt = x(t_0) \quad ; \; t_1 \le t_0 \le t_2}$

So, $\boxed{\int_{-\infty}^{+\infty} e^{-t}\cdot u(t)\cdot\delta(t-1)\,dt}$

$= e^{-1}\cdot u(1)$ $\quad(\because t_0 = 1$ & $x(t)=e^{-t}\cdot u(t)$.
& $-\infty \le 1 \le +\infty$).

$= e^{-1}$

(c) $e^{-t}\cdot u(t) * \delta(t-1)$.

$\Rightarrow$ It is the ~~Pro~~ Convolution property of Impulse.

i.e. $\boxed{x(t) * \delta(t-t_0) = x(t-t_0)}$.

So, $e^{-t} \cdot u(t) * \delta(t-1)$.

$$= e^{-(t-1)} \cdot u(t-1).$$

$\boxed{P\ 2.1.7}$ Let $x(t) = u(t-3) - u(t-5)$ & $h(t) = e^{-3t} \cdot u(t)$. Find $\frac{d}{dt} x(t) * h(t)$.

Soln:

$$x(t) = u(t-3) - u(t-5).$$

$$\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5).$$

~~Now,~~ $\cancel{\frac{d}{dt} x(t) * h(t)} = \cancel{\{[\delta(t-3) - \delta(t-5)]\} *}$
$\cancel{[u(t-3) - u(t-5)]}$.

$= \cancel{u(t-3) * \delta(t-3) - u(t-5) * \delta(t-5)}$
$\cancel{- u(t-5) * \delta(t-3) + u(t-5) * \delta(t-5)}$.
$= \cancel{u(t-6) - u(t-10) - u(t-8) + u(t-10)}$.

Now, $\frac{d\,x(t)}{dt} * h(t)$.

$$= [\delta(t-3) - \delta(t-5)] * e^{-3t} \cdot u(t).$$

$$= e^{-3t} \cdot u(t) * \delta(t-3) - e^{-3t} \cdot u(t) * \delta(t-5)$$

$$= e^{-3(t-3)} \cdot u(t-3) - e^{-3(t-5)} \cdot u(t-5).$$

Note: In LTI system, whatever happens in i/p same thing happens in o/p.

i.e. $\frac{d}{dt} x(t) * h(t) = \frac{d}{dt} . y(t)$.

=> In general,

$$\boxed{\begin{gathered} x^{m}(t) * h^{n}(t) = y^{m+n}(t). \\ m, n \rightarrow \text{order of differentiation} \end{gathered}}$$

=> $\boxed{x(t-\alpha) * h(t-\beta) = y(t-\alpha-\beta).}$

i.e. whatever be the delay in input same delay will occur in o/p as it is time Invariant system.

=> **Area:**

		Area
$x(t)$	$\longrightarrow$	A_x
$h(t)$	$\longrightarrow$	A_h.
$y(t)$	$\longrightarrow$	$A_x . A_h$.

Proof:

$$y(t) = \int x(\tau) . h(t-\tau) \, d\tau.$$

$$\text{Area } A_h = \int y(t) . dt = \iint x(\tau) . h(t-\tau) . d\tau . dt$$

$$= \int x(\tau) . d\tau \cdot \int h(t-\tau) . dt.$$

$\therefore \boxed{A_n = A_x \cdot A_n.}$ ✓

⇒ **Scaling:**

$$\boxed{x(\alpha t) * h(\alpha t) = \frac{1}{|\alpha|} \cdot y(\alpha t).}$$ ✓

Note:

⇒ In above case, property is valid only when the scaling factor in x & h should be same.

i.e. if $x(\alpha t) * h(\beta t) = ?$

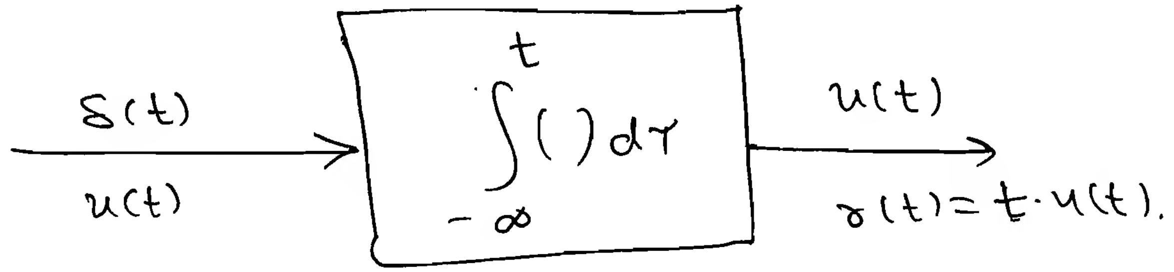
& $\alpha \neq \beta$

then No Comment.

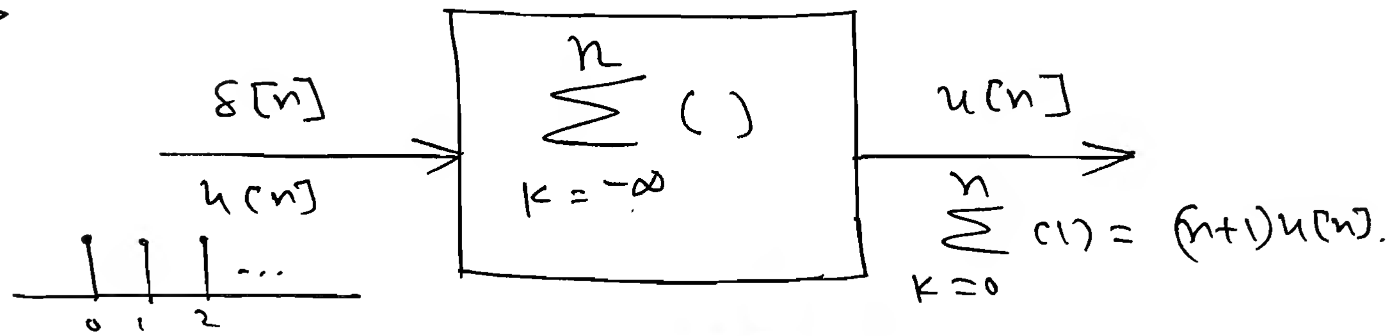
*

$$\boxed{\begin{aligned} u(t) * u(t) &= t \cdot u(t) \\ u[n] * u[n] &= (n+1)u[n]. \end{aligned}}$$

⇒



⇒



Q Convolution of $u[n]$ with $u[n-4]$ at $n=5$ is ___ ?

Sol^n:

$$y[n] = u[n] * u[n-4]$$

$$= (n+1-4)\,u[n] = \underline{(n+1-4)\,u[n]}$$

$$\therefore y[5] = (5+1-4]$$

$$\therefore \boxed{y[5] = 2}$$

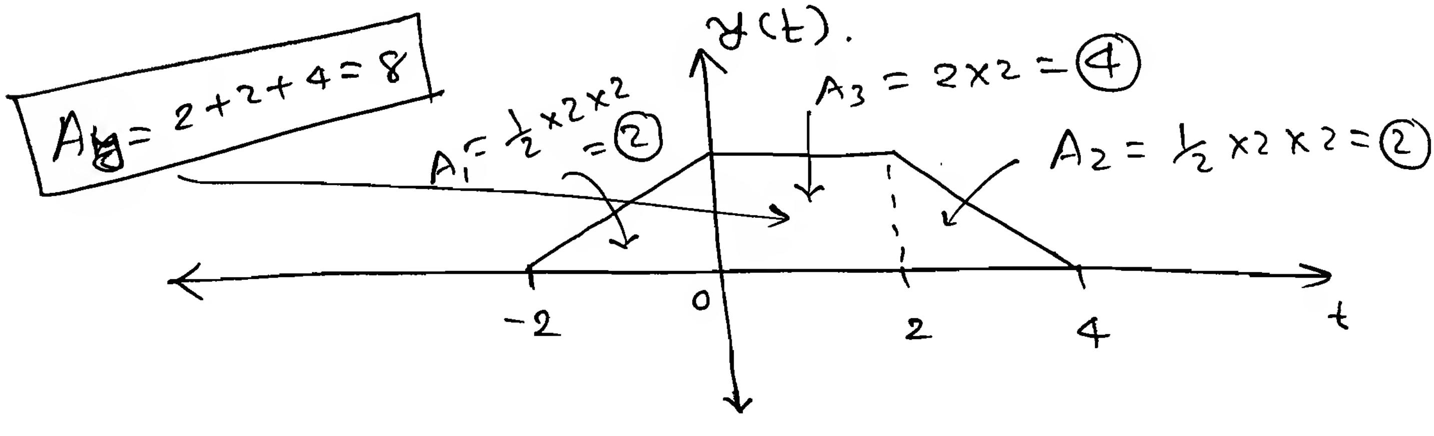
Q $[u(t+1) - u(t-3)] * [u(t+1) - u(t-1)] =$ ___

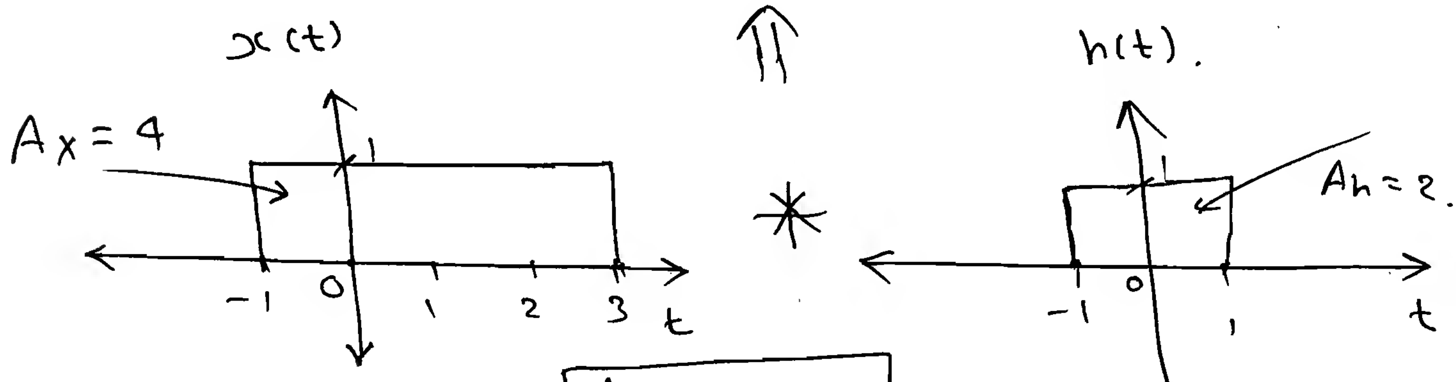
Sol^n:

$$y(t) = u(t+1) * u(t+1) - u(t+1) * u(t-1) - u(t-3) * u(t+1) + u(t-3) * u(t-1)$$

$$= r(t+1+1) - r(t+1-1) - r(t-3+1) + r(t-3-1)$$

$$= r(t+2) - r(t) - r(t-2) + r(t-4)$$





So, $\boxed{A_y = A_x \cdot A_h}$

Note: → Convolution of two unequal length rectangular functions is a Trapezium.

→ If length is same then it is a triangle.

Note:- → Convolution of the function into it self is equal to integration of that f^n. This statement is valid only for unit step f^n, $u(t)$.

i.e.
$$\boxed{x(t) * x(t) = \int_0^t x(\tau)\,d\tau. \quad \text{where, } x(\tau) = u(\tau).}$$

⇒ i.e.
$$\boxed{u(t) * u(t) = r(t) = t \cdot u(t) = \int_0^t u(\tau)\,d\tau.}$$

* Convolution using Differentiation:-

⇒

$$y(t) = x(t) * h(t).$$

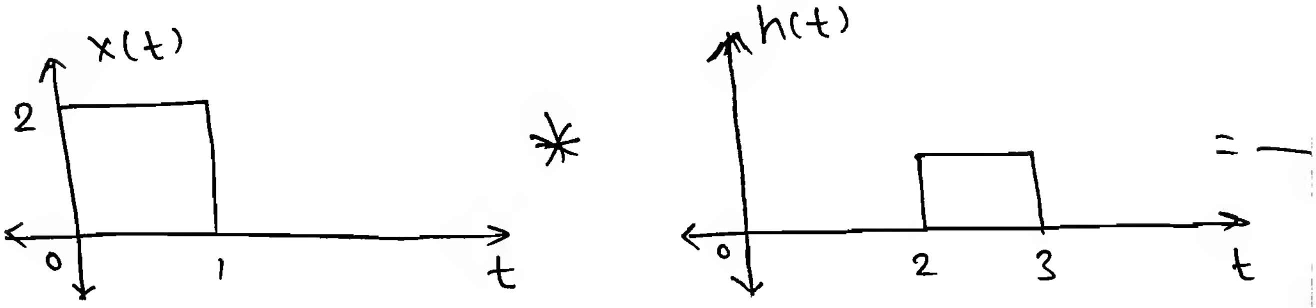
$$\frac{dy(t)}{dt} = \frac{dx(t)}{dt} * h(t)$$

(OR)

$$\therefore \frac{dy(t)}{dt} = x(t) * \frac{d}{dt} h(t).$$

$$\Rightarrow \boxed{y(t) = \int_{-\infty}^{t} \left[x(\tau) \cdot \frac{d}{d\tau} h(t)\right] d\tau.}$$

a



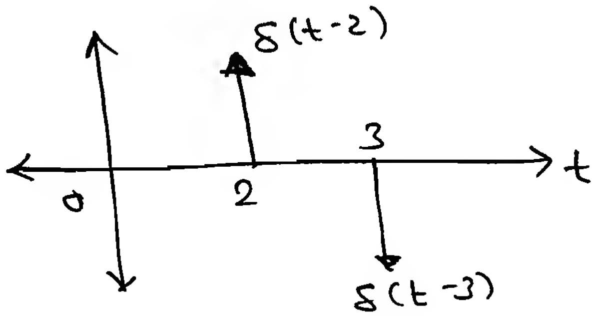
Sol^n:

$$x(t) = u(t) - u(t-1).$$

$$h(t) = u(t-2) - u(t-3).$$

$$\Rightarrow \frac{d}{dt} \cdot h(t) = \delta(t-2) - \delta(t-3).$$

$$=$$



Now, $y(t) = x(t) * h(t).$

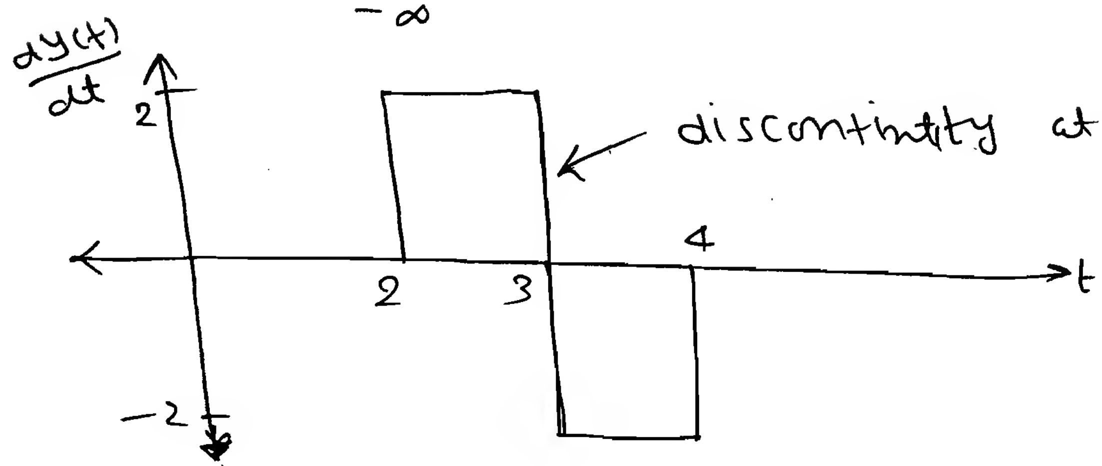
$$\Rightarrow \frac{dy(t)}{dt} = x(t) * \frac{dh(t)}{dt}.$$

$$= x(t) * \frac{d}{dt}\left[u(t-2) - u(t-3)\right].$$

$$= x(t) * \delta(t-2) - x(t) * \delta(t-3).$$

$$\frac{dy(t)}{dt} = x(t-2) - x(t-3).$$

$$\Rightarrow \quad y(t) = \int_{-\infty}^{t} [x(t-2) - x(t-3)]\,dt.$$



$$\Rightarrow \quad y(t) = \int_{-\infty}^{t} [x(t-2) - x(t-3)]\,dt.$$

Case-(i): $\quad 2 \leq t < 3.$

$$y(t) = \int_{2}^{t} 2\,dt = 2(t-2).$$

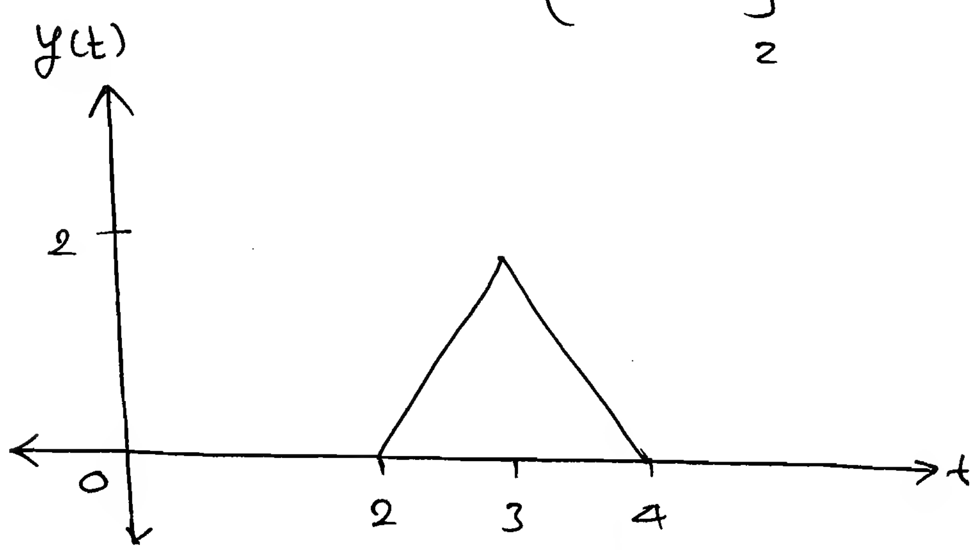
$t=3$

Case-(ii): $\quad 3 \leq t < 4$

$$y(t) = 2 + \int_{3}^{t} (-2)\,dt = -2t + 8.$$

$$\left(\because \int_{2}^{3} 2\,dt + \int_{3}^{t} -2\,dt. \right)$$

So,



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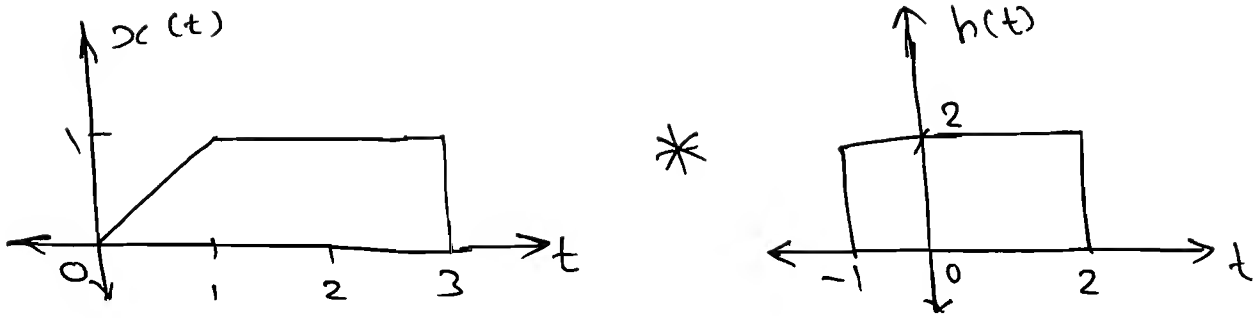


Sol^n:

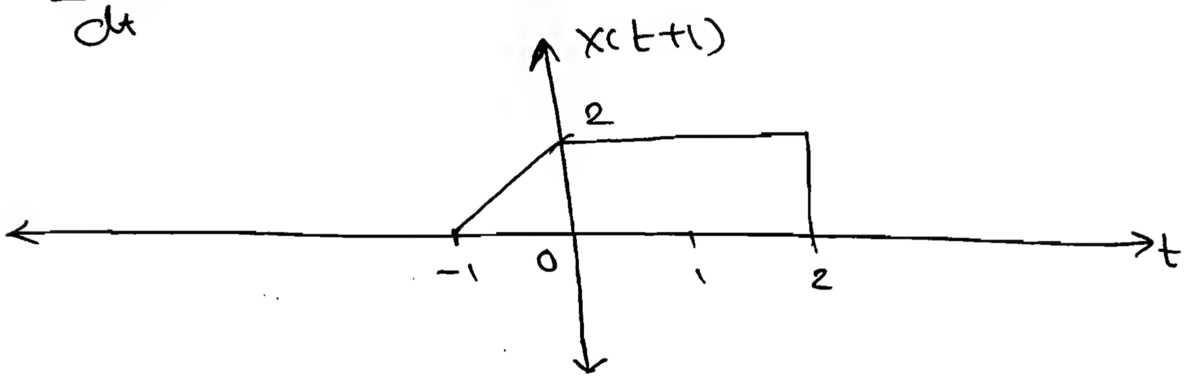
$$y(t) = x(t) * h(t).$$

$$\frac{dh(t)}{dt} = 2\,\delta(t+1) - 2\,\delta(t-2).$$

$$\frac{dy(t)}{dt} = x(t) * \frac{dh(t)}{dt}.$$

$$= x(t) * [2\,\delta(t+1) - 2\,\delta(t-2)].$$

$$\therefore \frac{dy(t)}{dt} = 2\,x(t+1) - 2\,x(t-2).$$



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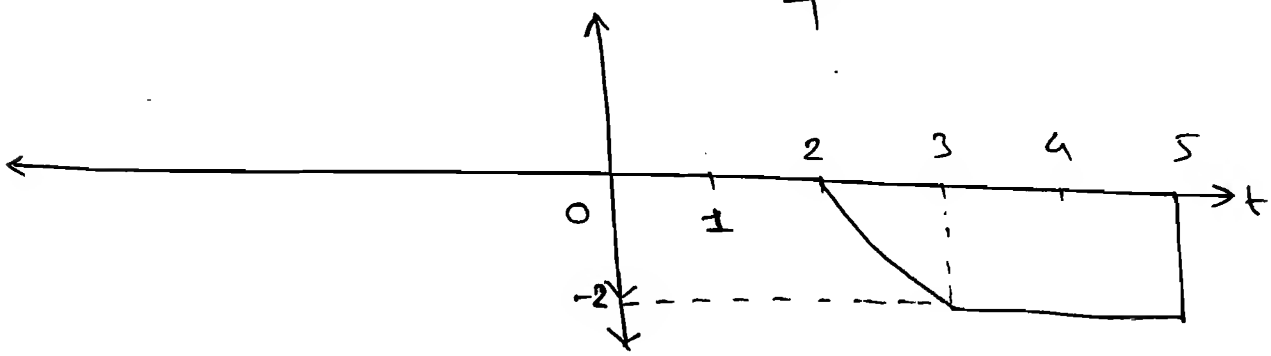

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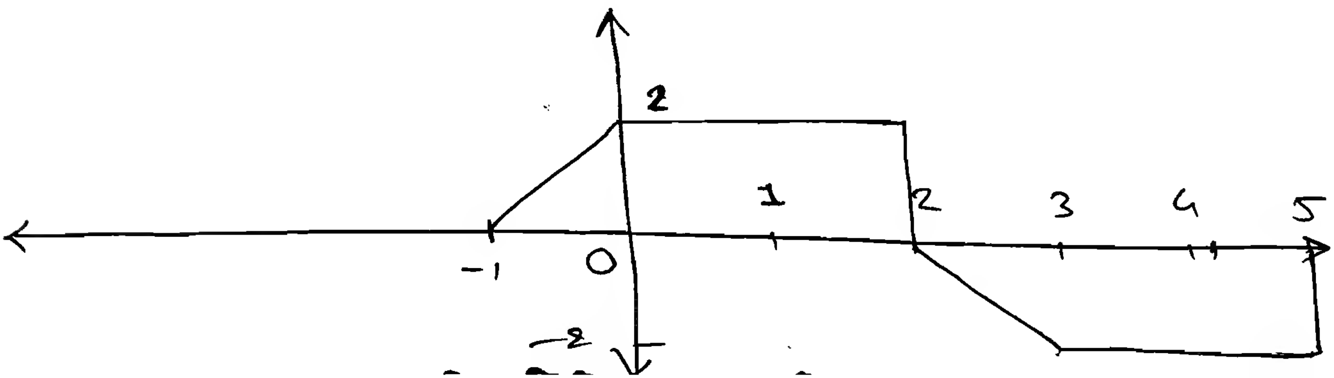

So, $y(t) = \int_{-\infty}^{t} \left(\frac{dy(t)}{dt}\right) dt$.

Now, <u>Case – (i)</u>: $-1 \leq t < 0$.

$$y(t) = \int_{-1}^{t} 2(t+1)\, dt$$

$$= \left[t^2 + t\right]_{-1}^{t} = t^2 + t - 1 + 1$$
$$= t^2 + t \;;\; -1 \leq t < 0.$$

<u>case – (ii)</u>: $0 \leq t < 2$.

$$y(t) = \int_{0}^{t} (2)\, dt = 2t \;;\; 0 \leq t < 2.$$

$t = 2$

as discon^t at $t = 2$.

<u>Case – (iii)</u>: $2 \leq t < 3$.

$$y(t) = \int_{2}^{t} (-2(t-2))\, dt + 4.$$

$$= \int_{2}^{t} (-2t + 4)\, dt + 4.$$

$$= 4 + \left[4t - t^2\right]_{2}^{t}$$

$$= \cancel{4} + 4t - t^2 - \cancel{8} + \cancel{4}.$$

$\therefore\; y(t) = 4t - t^2 \;;\; 2 \leq t < 3.$

<u>Case – (iv)</u>: $3 \leq t < 5$.

$$y(t) = \int_{3}^{t} (-2)\, dt = -2\,(t-3) = 6-t. \quad ; 3 \le t < 5$$

$$\therefore \; y(t) = t^2 + t \quad ; \; -1 \le t < 0.$$

$$= 2t \quad ; \; 0 \le t < 2.$$

$$= 4t - t^2 \quad ; \; 2 \le t < 3.$$

$$y(t) = 6 - t \quad ; \; 3 \le t < 5.$$

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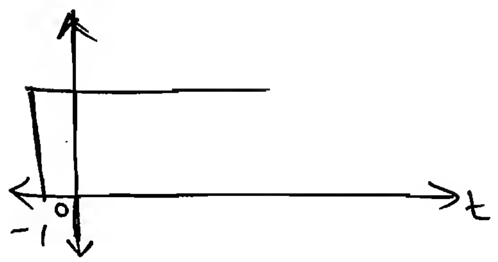

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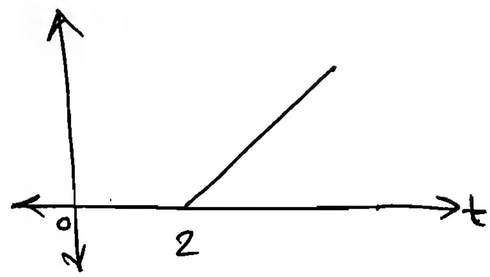


$= \_\_\_\_.$

Soln:

Assume, $g(t) = x(t) * h(t).$

Let, $x(t) = u(t).$

$h(t) = r(t).$

$$g(t) = u(t) * r(t).$$

$$\therefore \; g'(t) = \delta(t) * r(t).$$

$$g'(t) = r(t) \quad (\because t_0 = 0).$$

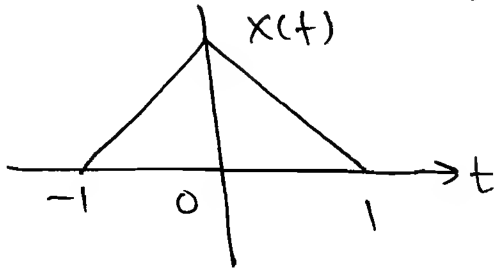
$$\therefore \; g(t) = \int_{0}^{t} t \cdot dt = t^2/2 \cdot u(t).$$

$$\therefore x(t+1) * r(t-2) = g(t+1-2).$$

$$= g(t-1).$$

$$= \frac{(t-1)^2}{2} \cdot u(t-1).$$

P 2.1.8 An Input signal $x(t)$ shown in figure is applied to the system with impulse response $h(t) = \sum_{n=-\infty}^{\infty} \delta(t-3n)$. Find the output.



Sol^n:

$$h(t) = \sum_{n=-\infty}^{+\infty} \delta(t-3n).$$

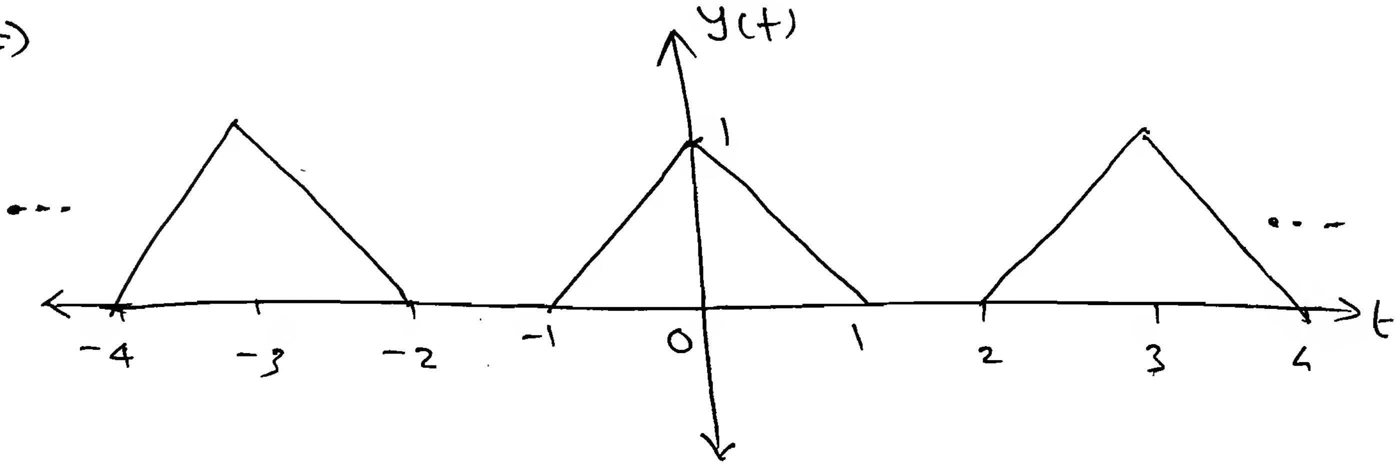
$$= \cdots + \delta(t+6) + \delta(t+3) + \delta(t) + \delta(t-3) + \delta(t-6) + \cdots$$

$$y(t) = x(t) * h(t).$$

$$= x(t) * (\cdots + \delta(t+6) + \delta(t+3) + \delta(t) + \delta(t-3) + \delta(t-6) + \cdots).$$

$$\therefore\ y(t) = \cdots + x(t+6) + x(t+3) + x(t) + x(t-3) + x(t-6) + \cdots$$

$\Rightarrow$



Note: Convolution of a general signal with periodic train of impulses is periodic repetation of general signal.

$\Rightarrow$ System length should be ~~the~~ more than the input signal length so that we can retain the information without distroying.

* Discrete Convolution:

P 2.1.11 Consider the signal $h(n) = \left[\frac{1}{2}\right]^{n-1}. \{u(n+3) - u[n-10]\}$.

Such that $h(n-k) = \begin{cases} \left(\frac{1}{2}\right)^{n-k-1} ; & A \leq k \leq B \\ 0 ; & \text{elsewhere} \end{cases}$ Find A & B?

Sol^n:

$$h(n) = \left[\frac{1}{2}\right]^{n-1} \cdot (1) \; ; \quad -3 \leq n \leq 9.$$

$$\therefore h(n-k) = \left[\frac{1}{2}\right]^{n-k-1} \; ; \quad -3 \leq n-k \leq 9$$

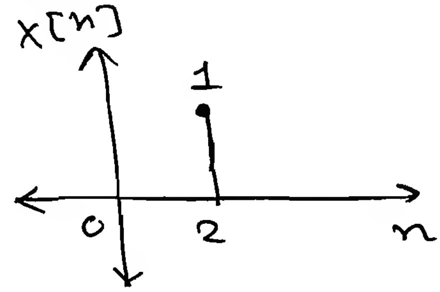
$$-3-n \leq -k \leq 9-n.$$

$$\therefore \boxed{n-9 \leq k \leq n+3.}$$

P 2.1.12 A linear system with Input $x(n)$ & output $y(n)$ related as

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k) . g(n-2k)$$ where

$g(n) = u(n) - u(n-4)$. Find $y(n)$ when $x(n) = \delta(n-2)$.



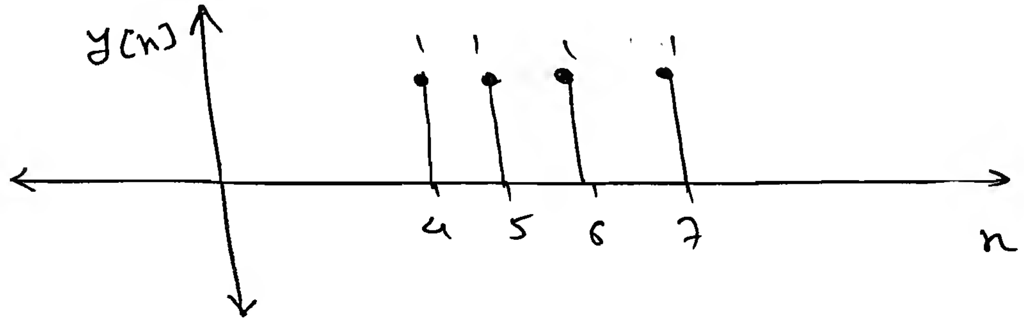
Sol^n: Here, $x[n] = \delta[n-2]$

$\Rightarrow x[2] = 1$

$\therefore y(n) = x(2) \cdot g(n-4)$.

$y[n] = g(n-4)$

$\therefore y[n] = u[n-4] - u[n-8]$.



$\rightarrow$ above system is time variant.

* Convolution Property of Discrete Impulse.

$\Rightarrow$ $$\boxed{x(n) * \delta(n-n_0) = x(n-n_0).}$$

P 2.1.14 Find the convolution of

$x(n) = \{1, 2, 3, 4\}$ & $h(n) = \{1, 2, 1, -1\}$.

Sol^n: There are total ~~five~~ four methods for convolution:

① Sliding strip.
② Array method
③ Sum by column.
④ ~~Polynomical~~ ~~division~~
⑤ Recursive Division.

① Sliding Strip method:

→

$h[k]$	1	2	1	-1
$x[-k]$ 4 3 2	1			

$y(0) = 1$

→

$h[k]$			1	2	1	-1
			×	×		
	4	3	2	1		

$y(1) = 2 + 2 = 4.$

→

$h[k]$		1	2	1	-1
		×	×	×	
	4	3	2	1	

$y(2) = 3 + 4 + 1 = 8$

→

$h[k]$	1	2	1	-1
	×	×	×	×
	4	3	2	1

$y(3) = 4 + 6 + 3 - 1$
$= 12.$

→

$h[k]$	1	2	1	-1		
		×				
		4	3	2	1	

$y(4) = 8 + 3 - 2 = 9$

→

$h[k]$	1	2	1	-1		
			4	3	2	1

$y(5) = 4 - 3 = 1.$

→

$h(k)$	1	2	1	-1			
				4	3	2	1

$y(6) = -4 = -4.$

$\therefore\ y(n) = \{1, 4, 8, 12, 9, 1, -4\}.$

② Array Method:

	1	2	3	4
1	1	2	3	4
2	2	4	6	8
1	1	2	3	4
-1	-1	-2	-3	-4

$y(n) = \{1, 4, 8, 11, 9, 1, -4\}.$

③ Sum by Column :-

$\Rightarrow$

$$
\begin{array}{rcccccccc}
x[n] = & 1 & 2 & 3 & 4 & & & \\
h[n] = & 1 & 2 & 1 & -1 & & & \\
\hline
 & 1 & 2 & 3 & 4 & & & \\
 & & 2 & 4 & 6 & 8 & & \\
 & & & 1 & 2 & 3 & 4 & \\
 & & & & -1 & -2 & -3 & -4 \\
\hline
\end{array}
$$

$$y[n] = \{1, 4, 8, 11, 9, 1, -4\}.$$

~~④ Polynomial Division method:~~

$\Rightarrow$ ~~$x(n) = \{1, 2, 3, 4\}$~~

~~$\omega^0 \quad \omega^1 \quad \omega^2 \quad \omega^3$~~

④ Recursive Division :-

$$y[n] = \sum x(n-k) \cdot h(k).$$

$\underline{n=0}$ $\quad y[0] = x(0) \cdot h(0).$

$\underline{n=1}$ $\quad y[1] = x[0] \cdot h[1] + x[1] \cdot h[0]$

$\underline{n=2}$ $\quad y[2] = x[0] \cdot h[0] + x[1] \cdot h[1] + x[2] \cdot h[0].$

* Deconvolution (system Identification):

Q i/p $x[n] = \{1, 2, 3, 4\}$

o/p $y[n] = \{1, 4, 8, 11, 9, 1, -4\}$.

$h[n] = ?$

Soln: Method-1: Sum by Column:

$\Rightarrow$

$x[n] \rightarrow$	1	2	3	4			
$h[n] \rightarrow$	a	b	c	d			
	a	2a	3a	4a			
		b	2b	3b	4b		
			c	2c	3c	4c	
				d	2d	3d	4d
$y[n] \rightarrow$	1	4	8	11	9	1	-4

$\Rightarrow \boxed{a=1}$, $\boxed{d=-1}$,

$2a+b=4$

$2+b=4$

$\Rightarrow \boxed{b=2}$

$3a+2b+c=8$

$3+4+c=8$

$\therefore \boxed{c=1}$

$$\boxed{h[n] = \{1, 2, 1, -1\}.}$$

Method-2: Recursive division:

$\Rightarrow \quad y[n] = \sum x[n-k] \cdot h[k].$

$n=0 \quad y[0] = x[0] \cdot h[0] \longrightarrow$ find $h(0)$

$n=1 \quad y[1] = x[0] \cdot h[1] + x[1] \cdot h[0] \rightarrow$ find $h[1]$.

$n = 2$ $\quad y[2] = x[0] \cdot h[2] + x[1]\,h[1] + x[2] \cdot h[0].$

Method - 3 : Polynomial division

$\rightarrow \quad x[n] = \{\underset{\omega^0}{1}, \underset{\omega^1}{2}, \underset{\omega^2}{3}, \underset{\omega^3}{4}\}.$

$\quad y[n] = \{\underset{\omega^0}{1}, \underset{\omega^1}{4}, \underset{\omega^2}{8}, \underset{\omega^3}{11}, \underset{\omega^4}{9}, \underset{\omega^5}{1}, \underset{\omega^6}{-4}\}.$

$$1 + 2\omega + \omega^2 - \omega^3$$

$$1 + 2\omega + 3\omega^2 + 4\omega^3 \;\big)\; 1 + 4\omega + 8\omega^2 + 11\omega^3 + 9\omega^4 + \omega^5 - 4\omega^6$$

$$1 + 2\omega + 3\omega^2 + 4\omega^3$$

$$2\omega + 5\omega^2 + 7\omega^3 + 9\omega^4 + \omega^5 - 4\omega^6$$

$$2\omega + 4\omega^2 + 6\omega^3 + 8\omega^4$$

$$0 + 1\omega^2 + 1\omega^3 + 1\omega^4 + \omega^5 - 4\omega^6$$

$$\omega^2 + 2\omega^3 + 3\omega^4 + 4\omega^5$$

$$-\omega^3 + 2\omega^4 + 6\omega^5 - 4\omega^6$$

$$\omega^3 - 2\omega^4 - 6\omega^5 + 4\omega^6$$

$$0 \quad 0 \quad 0 \quad 0$$

So, $\boxed{h[n] = \{1, 2, 1, -1\}.}$

* ordinary convolution | Periodic (or) circular convolution.

Ordinary convolution	Periodic (or) circular convolution
① $x_1[n] \rightarrow N_1$ Samples.	① $x_p[n] \rightarrow N_1$
② $x_2[n] \rightarrow N_2$ Samples.	② $h_p[n] \rightarrow N_2$
$y[n] \rightarrow (N_1+N_2-1)$.	$y_p[n] \rightarrow \max.(N_1, N_2)$

Q Find periodic (or) circular convolution of $x_p[n] = \{1\ 2\ 3\ 4\}$ & $h_p[n] = \{1\ 2\ 1\ -1\}$.

Soln:

$$x_p[n] = \quad 1 \quad 2 \quad 3 \quad 4$$
$$h_p[n] = \quad 1 \quad 2 \quad 1 \quad -1$$

ordinary conv. $= \underbrace{1 \quad 4 \quad 8 \quad 11} \quad 9 \quad 1 \quad -4$

wrap around last half of $y[n]$ $\quad + \quad 9 \quad 1 \quad -4 \quad 0$

$$10 \quad 5 \quad 4 \quad 11$$

Periodic conv. o/p $y[n]$.

Note: Linear conv. + Aliasing = Periodic Conv.

⇒ Matrix method:

$\{1\ 2\ 3\ 4\ 1\ 2\ 3\ 4\ 1\ 2\ 3\ 4\ 1\ 2\ 3\ 4\}$

$$\underset{4\times4}{\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ \underline{\underline{4}} & \underline{\underline{3}} & \underline{\underline{2}} & 1 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 1 \\ 2 \\ 1 \\ -1 \end{bmatrix}} = [10\ 5\ 4\ 11]$$

$\boxed{P\ 2.1.17}$ Given $x = [a, b, c, d]$ as the input to an LTI system produces an output $y = [x, x, x, x, \ldots$ repeated N times]. The impulse response of the system is ______.

Sol^n:

$$y = [\underbrace{a\ b\ c\ d}_{\delta(n)}\ \underbrace{a\ b\ c\ d}_{\delta(n-4)}\ \underbrace{a\ b\ c\ d}_{\delta(n-8)} \ldots N \text{ times}].$$

$\therefore$

$$\boxed{\delta = \sum_{n=0}^{N-1} \delta(n-4i)}$$

$\boxed{Q}$ Given $y(t) = e^{-t}.u(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k)$ such that $y(t) = Ae^{-t}$ for $0 \le t < 3$ find A?

Sol^n:

$$y(t) = e^{-t}.u(t) * [\cdots + \delta(t+6) + \delta(t+3) + \delta(t) + \delta(t-3) + \delta(t-6) + \cdots].$$

So, $y(t) = \cdots + e^{-(t+6)} u(t+6) + e^{-(t+3)} u(t+3)$

$+ e^{-t}.u(t) + e^{-(t-3)}.u(t-3) + e^{-(t-6)}.u(t-6) + \cdots$

as $y(t) = Ae^{-t}$ for $0 \le t < 3$

$$y(t) = Ae^{-t} = \cdots + e^{-(t+6)} + e^{-(t+3)} + e^{-t}.$$

$$A\cancel{e^{-t}} = \cancel{e^{-t}}(1 + e^{-3} + e^{-6} + \cdots]$$

$$A = 1 + e^{-3} + e^{-6} + e^{-9} + \cdots$$

$$\therefore \boxed{A = \frac{1}{1 - e^{-3}}} \qquad \left(\because A_n = \frac{a}{1-r}\right).$$

P2.1.18 Suppose for an L.T.I. system if the input applied is $\delta(n) - \delta(n-1)$, the output is observed to be $\delta(n) - \delta(n-1) + 2\delta(n-3)$. Find the output due to the input $7\delta(n) - 7\delta(n-2)$?

Soln:

i/p	o/p
$\delta(n) - \delta(n-1)$	$\delta(n) - \delta(n-1) + 2\delta(n-3)$.
$7\delta(n) - 7\delta(n-2)$	

Let, $x_1(n) = \delta(n) - \delta(n-1)$.

$x_1(n-1) = \delta(n-1) - \delta(n-2)$.

$\therefore x_1(n) + x_1(n-1) = \delta(n) - \delta(n-2)$.

so, $x(n) \longrightarrow \delta(n) - \delta(n-1) + 2\delta(n-3)$.

$$y_2(n) = 7y_1(n) + 7y_1(n-1).$$

$$= 7\delta(n) - 7\cancel{\delta(n-1)} + 14\delta(n-3) + 7(\cancel{\delta(n-1)}) - 7\delta(n-2) + 14\delta(n-4).$$

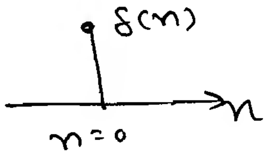
$$\therefore \boxed{y_2(n) = 7\delta(n) - 7\delta(n-2) + 14\delta(n-3) + 14\delta(n-4)}$$

* Properties of LTI Systems:

① **Causal:**

⇒ Before the application of input ~~the~~ ~~output~~ as impulse at $n=0$. We can not get the output as impulse response before $n=0$. i.e. $\boxed{h(n)=0 \text{ for } n<0.}$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t).$$



$\Rightarrow$ O/P $h(n) =$ I.Response
$= 0 \quad ; n<0.$

② **Stable:**

$$\Rightarrow \int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty = \text{finite.}$$

$$\sum_{n=-\infty}^{+\infty} |h(n)| < \infty.$$

Proof:

$$|y(t)| = \int_{-\infty}^{\infty} |x(t-\tau).\,h(\tau)|\, d\tau$$

B.I.B.O.

$$= \int_{-\infty}^{\infty} \underbrace{|x(t-\tau)|}_{\text{finite}} \cdot |h(\tau)| \cdot d\tau$$

$$= \text{finite}. \underbrace{\int_{-\infty}^{+\infty} |h(\tau)|\, d\tau}_{\text{finite}}.$$

$$\therefore \int_{-\infty}^{+\infty} |h(\tau)|\, d\tau < \infty = \text{finite.}$$

$\therefore$ Area under absolute value of impulse response is finite.

③ Static (or) Memory less:

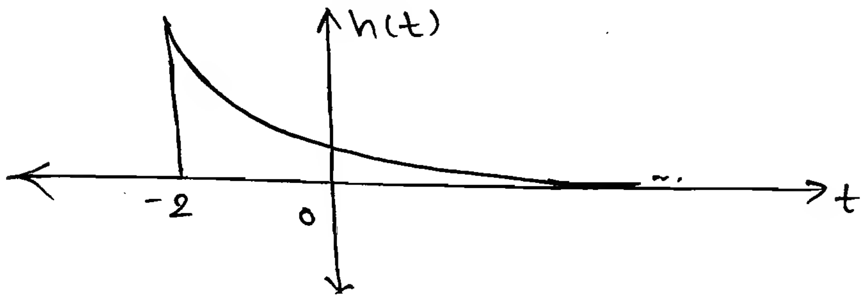
$$h(t) = K\,\delta(t).$$

(or) $h(t) = 0$, for $t \neq 0$.

Note: All Property indicate internal behaviour of the system.

$\boxed{Q}$ $h(t) = e^{-t}.\, u(t+2).$

Soln: $h(t) = e^{-t}.\, u(t+2).$



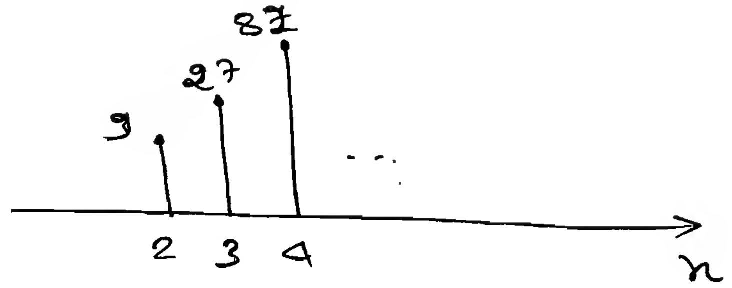
$h(t) \neq 0$ for $t<0$. So, Non-Causal.

Stability: $\int_{-2}^{\infty} e^{-t}.\, dt$ = finite = Stable.

$\boxed{Q}$ $h[n] = 3^n.\, u[n-2].$

Soln: $h[n] = 3^n$; $n \geq 2$.

$\Rightarrow$



$$\Rightarrow \quad \sum_{n=2}^{\infty} 3^n = 9 + 27 + 81 + \dots = \infty.$$

$$= \text{Unstable}.$$

$\rightarrow$ $h(n)$ is define for positive index, n

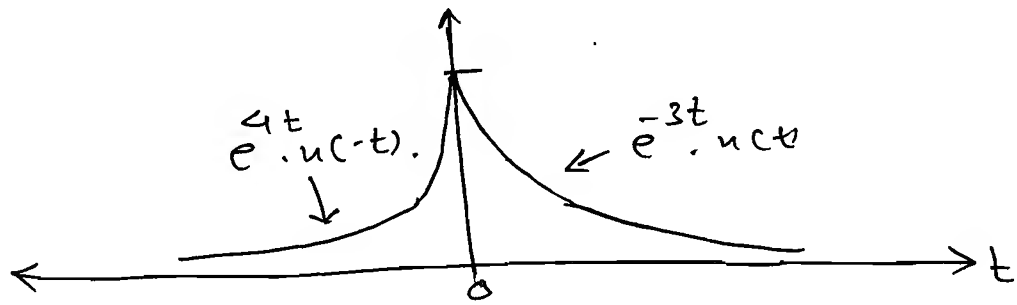
$\Rightarrow$ causal.

$\boxed{\text{P2.2.4.}}$ The range of a and b for the impulse response $h(n) = \begin{cases} a^n & ; n \geq 0. \\ b^n & ; n < 0. \end{cases}$ to be stable is ______.

Soln: $\boxed{\text{Q - 2.2.5}}$ Given $h(t) = e^{\alpha t} \cdot u(t) + e^{\beta t} \cdot u(-t)$. Find α & β for stable.

$$h(t) = e^{\alpha t} \cdot u(t) + e^{\beta t} \cdot u(-t).$$

i.e.



$$\int_{-\infty}^{\infty} |h(t)| \, dt.$$

$$= \int_{-\infty}^{0} e^{4t} \, dt + \int_{0}^{\infty} e^{-3t} \, dt.$$

$$= \left[\frac{e^{4t}}{4} \right]_{-\infty}^{0} + \left[\frac{e^{-3t}}{-3} \right]_{0}^{\infty}.$$

$= \frac{1}{4} + \frac{1}{3}$

$= 7/12 \; = \text{finite.}$

Sol^n of $\boxed{\text{P } 2.2.4.}$:

for stable: $\sum_{n=-\infty}^{\infty} h(n) = \text{finite} < \infty.$

$$= \sum_{n=-\infty}^{-1} b^n + \sum_{n=0}^{\infty} a^n.$$

① $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad \boxed{|a| < 1.}$ ✓

② $\sum_{n=-\infty}^{-1} b^n$, let $-n = m$

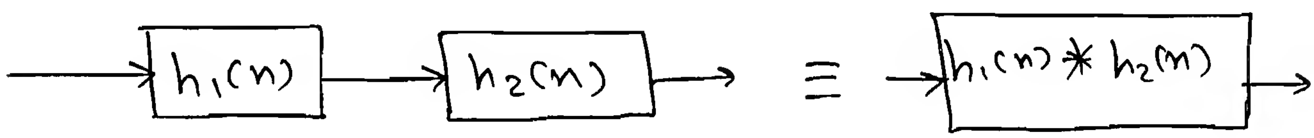
$$\therefore \Rightarrow \sum_{m=1}^{\infty} b^{-m} = 1 + b^{-1} + b^{-2} + b^{-3} + \cdots - 1.$$

$$= \frac{1}{1-b^{-1}} - 1, \quad |b^{-1}| < 1.$$

$$\Rightarrow \boxed{|b| > 1}$$ ✓

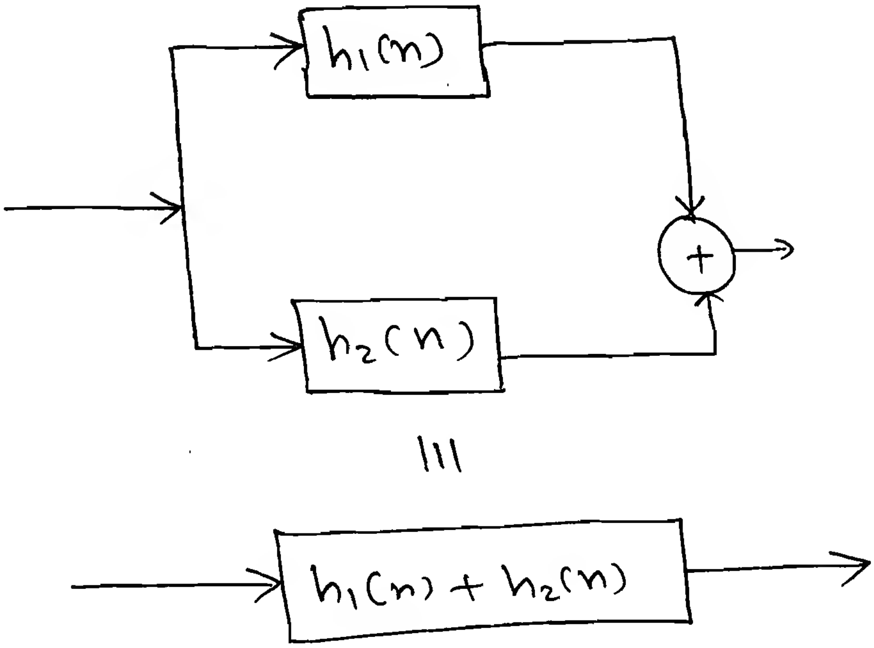
* ~~Series~~ 1.

* **Blocks in series:**



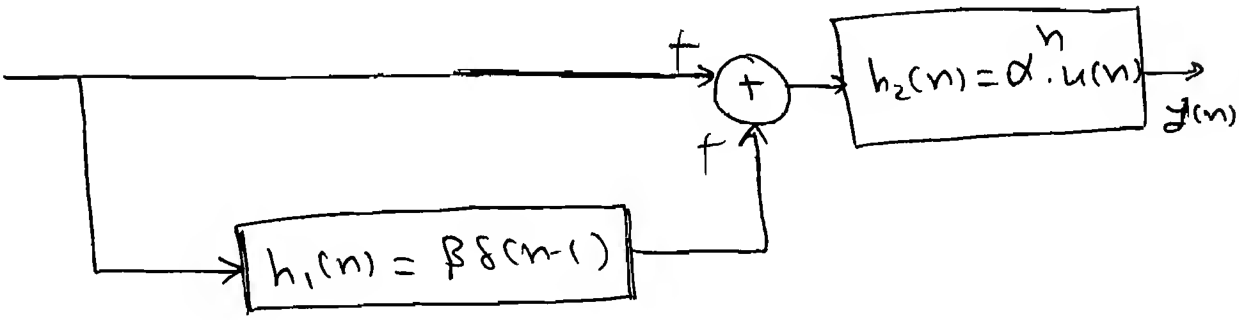
* Blocks in parallel:

⇒



⇒

$\boxed{P2.2.3}$ Consider the system in figure.



(a) Find. I.R. of overall system.

Soln:

$$h(n) = [\delta(n) + \beta\,\delta(n-1)] * [\alpha^n .\, u(n)].$$

$$= \delta(n) * \alpha^n .\, u(n) \quad + \quad \beta\, \delta(n-1) * \alpha^n .\, u(n).$$

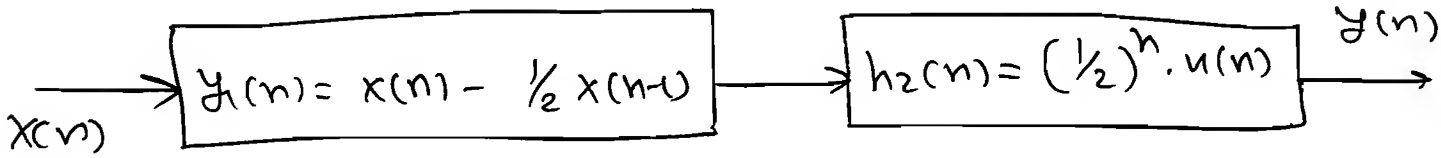
$$\boxed{h(n) = \alpha^n .\, u(n) + \beta .\, \alpha^{n-1} .\, u(n-1).}$$

(b) Is this system causal? Under what condn the system is stable.

⇒ yes, System is causal.

⇒ If $|\alpha| < 1$ then system is stable.

$\boxed{P\ 2.2.6}$ For the interconnected system shown in fig. find the overall impulse response.



Sol^n: for impulse response $x(n) = \delta(n)$.

$$\therefore h(n) = \left[\delta(n) - \frac{1}{2}\delta(n-1)\right] * \left[(1/2)^n . u(n)\right].$$

$$= \delta(n) * \left(\frac{1}{2}\right)^n . u(n) - \frac{1}{2}\delta(n-1) * \left(\frac{1}{2}\right)^n . u(n).$$

$$= \left(\frac{1}{2}\right)^n . u(n) - \frac{1}{2} . \left(\frac{1}{2}\right)^{n-1} . u(n-1).$$

$$= \left(\frac{1}{2}\right)^n . [u(n) - u(n-1)].$$

$$= \left(\frac{1}{2}\right)^n . \delta(n).$$

$$= \left(\frac{1}{2}\right)^0 . \delta(n).$$

$$= \delta(n).$$

④ <u>Invertible</u> :

$$\therefore \boxed{h(n) * h_{inv}(n) = \delta(n).}$$

P 2.2.2 Consider the D.T. system S_1 ~~is~~ , with I.R. $h(n) = (1/5)^n \cdot u(n)$.

(a) Find 'A' such that ~~h(n) =~~

$$h(n) - A h(n-1) = \delta(n).$$

Soln:

$$h(n) - A h(n-1) = \delta(n).$$

$$\therefore \left(\frac{1}{5}\right)^n \cdot u(n) - A \cdot \left(\frac{1}{5}\right)^{n-1} \cdot h(n-1) = \delta(n).$$

$\therefore$ A should $1/5$ so that

$$\left(\frac{1}{5}\right)^n [u(n) - u(n-1)] = \delta(n).$$

$$\therefore \boxed{A = 1/5}$$

(b) using result from part-(a), determine the I.R. $g(n)$ of an LTI system S_2 which is inverse of S_1.

Soln:

$$h(n) * g(n) = \delta(n)$$

$$\downarrow$$

$$g(n) = h_{Inv}(n).$$

$\therefore$ here $h(n) - A h(n-1) = \delta(n)$.

$$h(n) * [\underbrace{\delta(n) - A\,\delta(n-1)}_{h_{inv} = g(n)}] = \delta(n).$$

$$\therefore \boxed{g(n) = \delta(n) - A\,\delta(n-1).}$$

$\Rightarrow$ $y[n] = n\, x[n]$

T.V.   Non. Invertible.

$\Rightarrow$ $y[n] = x[n] \cdot x[n-3]$.

NL   N.I.

$\Rightarrow$ $y[n] = x[n] \cdot \sin\left[\frac{5\pi n}{6}\right]$.

T.V.   N.L   N.I.

$\Rightarrow$ $y(n) = \sum_{k=-\infty}^{+\infty} x(k)$.

L.T.I. System. $\Rightarrow$ Inverse is $y[n] - y[n-1]$.

Note: Impulse response term is valid only for LTI system.

$\rightarrow$ $x(k) = \delta(k) \Rightarrow y(n) = h(n)$.

if $h(n) = u(n)$.

$h(n) * h_{inv}(n) = \delta[n]$.

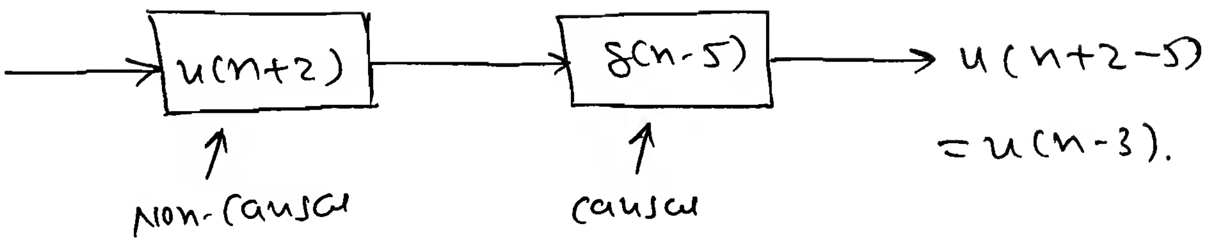
$\therefore u[n] * \underbrace{[\delta[n] - \delta[n-1]]}_{h_{inv}(n)} = \delta[n]$.

$\boxed{P2.2.7.}$ Determine whether each of the following statements are TRUE (or) FALSE.

① The cascade of a non causal LTI system with causal one is necessarily

non Causal.

Soln:



there is Chance of Causality.

So, Statement is false.

(2) If an LTI system is causal, it is stable.

Soln: let, $h(t) = u(t) \rightarrow$ LTI & causal.

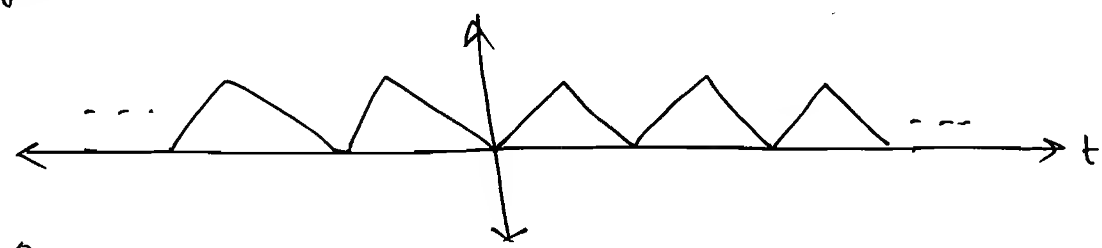
But $\int_{-\infty}^{\infty} |u(t)| = \int_{0}^{\infty} (1)\, dt = \infty$

= unstable.

So, False statement

(3) If h(t) is the I.R. of an LTI system which is periodic & non-zero, the system is unstable.

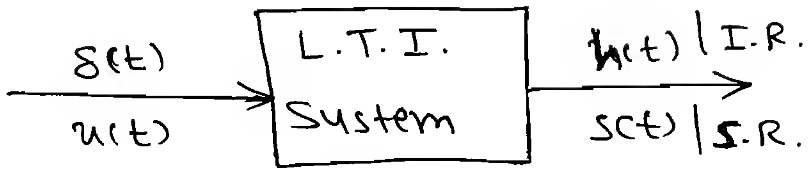
Soln: All periodic signals are ever lasting signal.



$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau = \infty \Rightarrow \text{Unstable.}$$

So, given Statement is True.

* Step Response :-



→ total internal behaviour of system is charu. by I.R.

→ Sudden change in system behaviour is char. by S.R.

$$\delta(t) = \frac{d}{dt} u(t).$$

$$\boxed{h(t) = \frac{d}{dt} s(t).}$$

&

$$\boxed{s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau.}$$

Q $x(t) = u(t-2)$ & $h(t) = e^{-3t} u(t)$.

Soln:

O/p $y(t) = s(t-2)$.

$$s(t) = \int_{-\infty}^{t} e^{-3\tau} . u(\tau)\, d\tau.$$

$$= \int_{0}^{t} e^{-3\tau} . d\tau.$$

$$= \left[\frac{e^{-3\tau}}{-3}\right]_0^t$$

$$s(t) = \frac{1 - e^{-3t}}{3}$$

$$\Rightarrow s(t-2) = y(t)$$

$$= \frac{1 - e^{-3(t-2)}}{3}.$$

Note: Whenever the one signal is step signal then we can find O/P by just integrating other signal.

Q 2.3.2. Find the impulse response of the system if the step response is $s(t) = \cos\omega_0 t\, u(t)$.

Sol^n:

I.R. $h(t) = \frac{d}{dt}(s(t))$.

$\therefore h(t) = \frac{d}{dt}[\cos\omega_0 t \cdot u(t)]$.

$\therefore h(t) = -\omega_0 \sin\omega_0 t \cdot u(t) + \cos\omega_0 t \cdot \delta(t)$.

$= -\omega_0 \sin\omega_0 t \cdot u(t) + [\cos(\omega_0 \cdot 0)] \cdot \delta(t)$

$\therefore h(t) = -\omega_0 \sin\omega_0 t \cdot u(t) + \delta(t)$.

P 2.3.4 If the unit step response of a system is $(1-e^{-\alpha t})u(t)$, then its unit impulse response is ____.

Sol^n: $s(t) = [1 - e^{-\alpha t}]u(t)$.

$\therefore h(t) = \frac{d}{dt} s(t)$.

$= \frac{d}{dt}[u(t) - e^{-\alpha t} \cdot u(t)]$.

$= \delta(t) - e^{-\alpha t} \cdot \delta(t) + \alpha e^{-\alpha t} \cdot u(t)$.

$$= \cancel{\delta(t)} - e^{-0} \cdot \cancel{\delta(t)} + \alpha \cdot e^{-\alpha t} \cdot u(t).$$

$$= \alpha \cdot e^{-\alpha t} \cdot u(t).$$

$\boxed{P2.3.1.}$ Find the step response of the System if the impulse response is

$$h(n) = (0.5)^n \cdot u(n).$$

Sol^n:

$$s(n) = \sum_{k=-\infty}^{\infty} h(k).$$

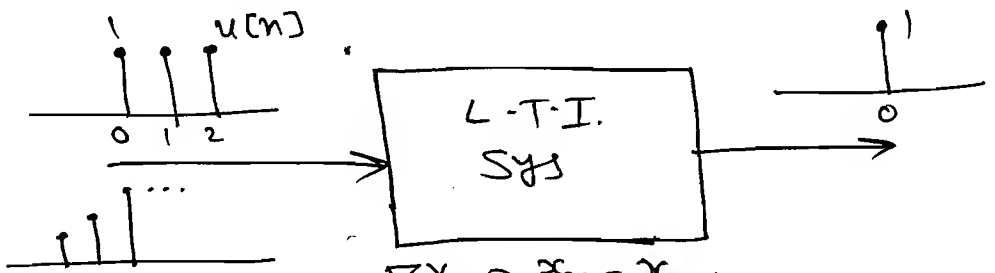
$$\therefore \quad s(n) = \sum_{k=-\infty}^{+\infty} (0.5)^k \cdot u(k).$$

$$= \sum_{k=0}^{n} (0.5)^k.$$

$$= 1 + (0.5) + (0.5)^2 + (0.5)^3 + \dots + (0.5)^n.$$

$$s(n) = \frac{1 - (0.5)^{n+1}}{1 - 0.5} \; ; \quad n \geq 0.$$

$\boxed{P2.3.3.}$ An LTI system with Input $u(n)$ Produces the output as $\delta(n)$, then find the output due to the input $n\,u(n)$?



$$\nabla x_n = x_n - x_{n-1}$$

$\therefore \quad \delta(n) = u(n) - u(n-1).$

$\therefore$

$$o/p = n u(n) - (n-1)\, u(n-1).$$

$$= n u(n) - n u(n-1) + u(n-1).$$

$$= n\,(u(n) - u(n-1)) + u(n-1).$$

$$= n\,\delta(n) + u(n-1).$$

$$= 0 + u(n-1).$$

$$\boxed{o/p = u(n-1).}$$

$\boxed{P2.35.}$ Find the overall impulse response for the interconnected system shown in figure?

Soln:

$$h(t) = \left[-\left[\left[h_2(t) + h_3(t)\right] * h_4(t)\right] + h_1(t)\right] * h_5(t).$$

☆ Fourier Series:

→ It is an approximation process where a non-sinusoidal waveform is converted to sinusoidal such that all the periodic signal are represented as in unique form.

→ Sinusoids and Complex exponents are eigen functions of LTI system.
(same o/p as input)

→ For a signal to have fourier series, orthogonality is must condition. (Two signal $x_1(t)$ and $x_2(t)$ are orthogonal

if $$\int_{t_1}^{t_2} x_1(t) \cdot x_2(t)\, dt = 0.$$

→ Means area under inner product inner product of two signal is zero.

Frequency \ Time	Continuous	Discrete
Continuous	C.T.F.T.	D.T.F.T.
Discrete	C.F.S.	D.F.S / D.F.T.

Aim: one Period $\longrightarrow$ many freq.

$\omega_0 \longrightarrow n\omega_0$.

→ There are 2 reasons for evaluating the F.S.

1. To obtain an expression for $f(t)$ that applies everywhere, rather than only over a single period.

2. To obtain phasor, which indirectly tell how much power is available at each harmonic of the waveform.

* Trignometric Fourier series (T.F.S.).

→ $0, \omega_0, 2\omega_0, 3\omega_0, \cdots$

$$x(t) = a_0 \cos(0.\omega_0 t) + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + a_3 \cos 3\omega_0 t + a_4 \cos 4\omega_0 t + \ldots + b_0 \sin(0.\omega_0 t) + b_1 \sin \omega_0 t + b_2 \sin(2\omega_0 t) + b_3 \sin 3\omega_0 t + \ldots$$

$$\therefore \boxed{x(t) = \underbrace{a_0}_{d.c.} + \underbrace{\sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin \omega_0 n t.}_{a.c.}}$$

$$\Rightarrow \int_0^T \cos\omega_0 nt\, dt = \int_0^T \sin\omega_0 nt.\, dt = 0.$$

$n \neq 0$.

$$\Rightarrow \int_0^T \sin\omega_0 mt.\ \sin\omega_0 nt.\, dt = T/2 \ ; \ m = n$$
$$= 0 \ ; \ m \neq n.$$

$$\Rightarrow \int_0^T \cos\omega_0 mt.\ \cos\omega_0 nt.\, dt = T/2 \ ; \ m = n$$
$$= 0 \ ; \ m \neq n.$$

$$\Rightarrow \int_0^T \cos\omega_0 mt.\ \sin\omega_0 nt.\, dt = 0 \ ; \ \forall\ m, n$$

$\Rightarrow$ By integrating both side of the eqn (1)

$$\int_0^T g(t).dt = \int_0^T a_0\, dt + \sum_{n=1}^{\infty} \cancel{\int a_n \cos\omega_0 nt\, dt}^{\,0} + \cancel{\int b_n \sin\omega_0 nt\, dt}^{\,0}$$

$$\therefore \int_0^T g(t).dt = a_0 . T.$$

$$\Rightarrow \boxed{a_0 = \frac{1}{T} \int_0^T g(t).dt} = \text{d.c. value (or) avg. value.}$$

$$\Rightarrow \boxed{a_0 = \frac{\text{Area of } g(t) \text{ over a period}}{\text{fundamental period}}}$$

⇒ Multiply both side of eqn - ① by $\cos \omega_0 nt$ & integrating both side of eqn - ① then we get,

$$\int_0^T g(t).\cos\omega_0 nt\, dt = \int_0^T \cancel{a_0.\cos\omega_0 nt.dt}^{\,0}$$
$$+ \sum_{n=1}^{\infty} \int_0^T \cancel{a_n.\cos\omega_0 nt.\cos\omega_0 nt dt}^{\,T/2 \cdot a_n}$$
$$+ \int_0^T \cancel{b_n.\cos\omega_0 nt.\sin\omega_0 nt.dt}^{\,0}$$

$$\therefore \int_0^T g(t).\cos\omega_0 nt\, dt = a_n.\left(\frac{T}{2}\right)$$

$$\therefore \boxed{a_n = \frac{2}{T} \times \int_0^T g(t).\cos\omega_0 nt.dt.}$$

* Polar form:

⇒
$$g(t) = d_0 + \sum_{n=1}^{\infty} d_n \cos(\omega_0 nt + \theta_n).$$
$$(\text{or})\ d_0 + \sum_{n=1}^{\infty} d_n \sin(\omega_0 nt + \theta_n).$$

fore convienent we will use cosine.

$$\therefore g(t) = d_0 + \sum_{n=1}^{\infty} d_n \cos(\omega_0 nt + \theta_n).$$

$$\therefore \; g(t) = d_0 + \sum_{n=1}^{\infty} d_n \cos \omega_0 n t \cdot \cos \theta_n - d_n \sin \omega_0 n t \cdot \sin \theta_n.$$

$a_n = d_n \cos \theta_n, \quad b_n = -d_n \cdot \sin \theta_n.$

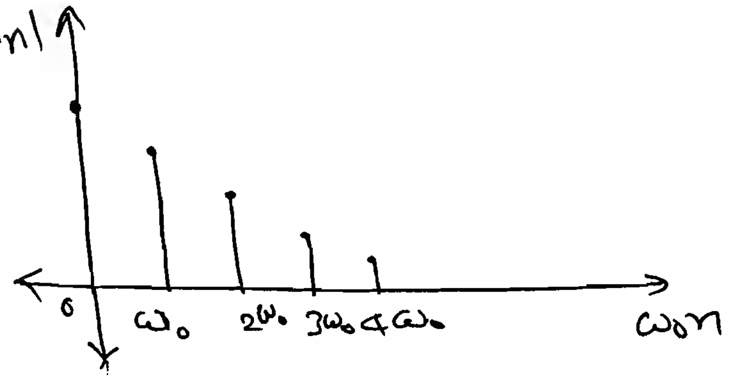
$a_0 = d_0.$

$$\underbrace{\boxed{\begin{aligned} \therefore\; |d_n| &= \sqrt{a_n^2 + b_n^2} \\ \therefore\; \theta_n &= \tan^{-1}\left(-\frac{b_n}{a_n}\right) \end{aligned}}}_{\text{Amplitude Spectrum.}}$$

$|d_n| = \sqrt{a_n^2 + b_n^2} \longrightarrow$ magnitude spectrum.

$\theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right) \longrightarrow$ phase spectrum.

$\Rightarrow$

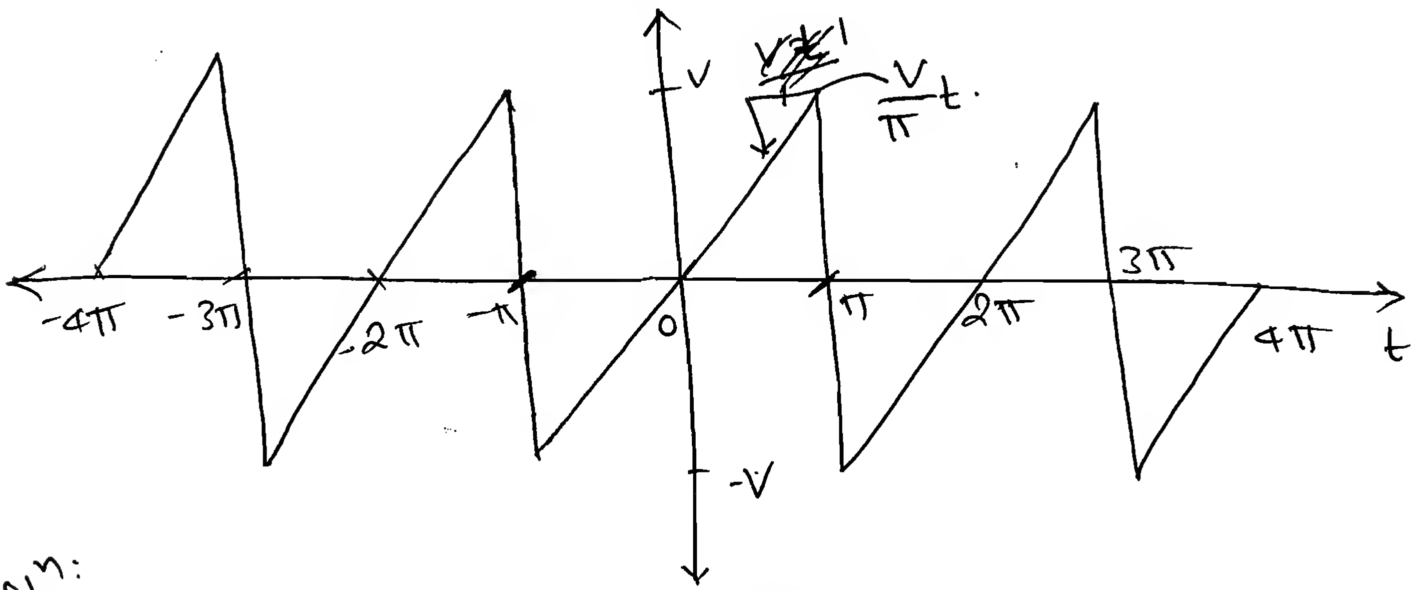


<u>Note</u>: From the magnitude spectrum we will inform that for what range of freq. the max. power is concentrated.

* <u>Effect of Symmetry on Fourier coefficient</u>:-

Symmetry	Condition	a_0	a_n	b_n
Even	$g(t) = g(-t)$	?	?	0
odd	$g(t) = -g(-t)$	0	0	?
Half-wave	$g(t) = -g(t \pm T/2)$	0	$= 0$; n-even $= ?$; n-odd	$= 0$, n-even $= ?$, n-odd

[a] Find the TFS representation for the periodic waveform shown below.



Soln:

$|\leftarrow T = 2\pi \rightarrow|$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1 \text{ rad/sec.}$$

→ Here, odd-symmetry.

So, $a_0 = 0$, $a_n = 0$.

$$x(t) = \frac{V}{\pi}t, \quad -\pi \le t \le \pi.$$

$$b_n = \frac{2}{T}\int_0^T x(t).\sin\omega_0 nt.\,dt$$

$$= \frac{\cancel{2}}{\cancel{2}\pi}\int_0^{2\pi} x(t).\sin\omega_0 nt.\,dt$$

$$= \frac{1}{\pi}.\int_{-\pi}^{\pi} x(t).\sin\omega_0 n t\,dt$$

$$= \frac{2}{\pi}\times\int_0^{\pi}\frac{V}{\pi}t.\sin\omega_0 nt\,dt.$$

$$= \frac{2V}{\pi^2} \times \left[(t)\cdot\left(-\frac{\cos\omega_0 nt}{\omega_0 n}\right) - (1)\left(-\frac{\sin\omega_0 nt}{(\omega_0 n)^2}\right) \right]_0^{\pi}$$

$\omega_0 = 1$ rad/sec.

$$= \frac{2V}{\pi^{\cancel{2}}} \times \left(-\frac{\cancel{\pi}}{n} \cdot \cos n\pi + 0 - 0 \right)$$

$$= \frac{2V}{\pi n} \times (-1) \times (-1)^n \qquad (\because \cos n\pi = (-1)^n).$$

$$b_n = \frac{2V}{\pi n} \cdot (-1)^{n+1}$$

$$\therefore \quad b_n = \frac{2V}{\pi n}, \quad n - \text{odd}.$$

$$= -\frac{2V}{\pi n}, \quad n - \text{even}.$$

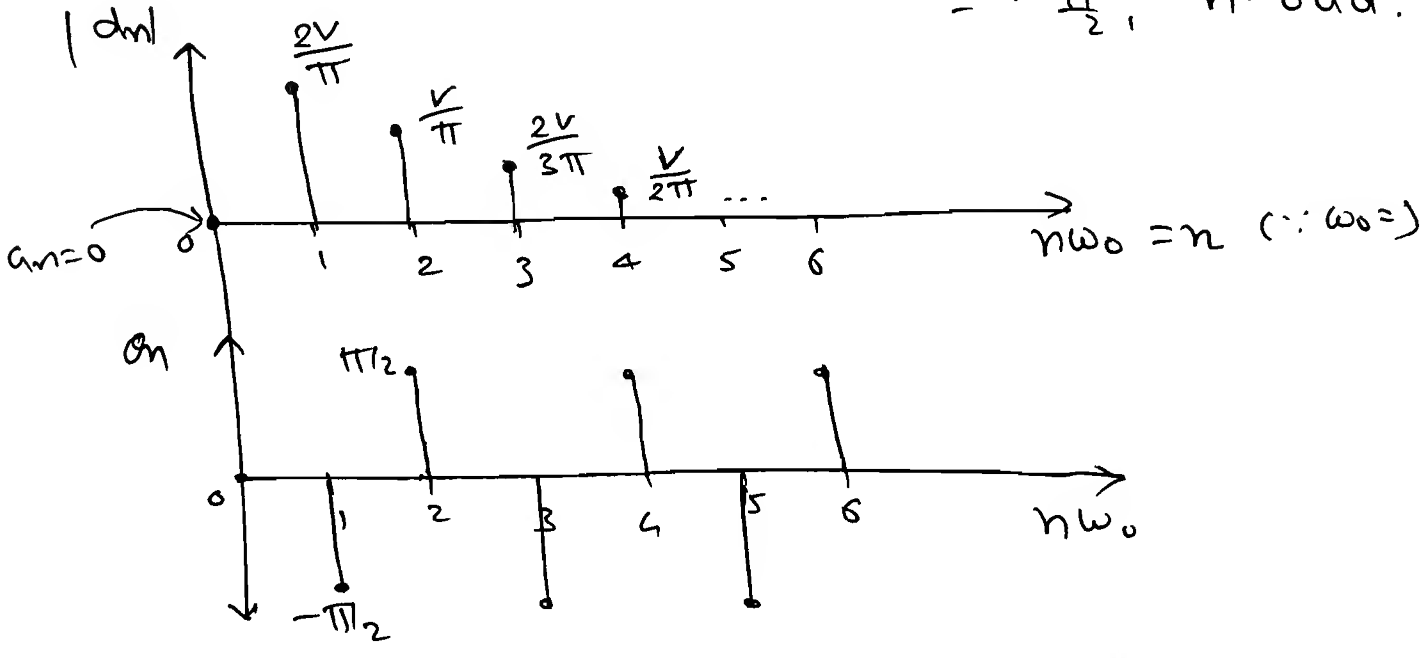
Polar form:

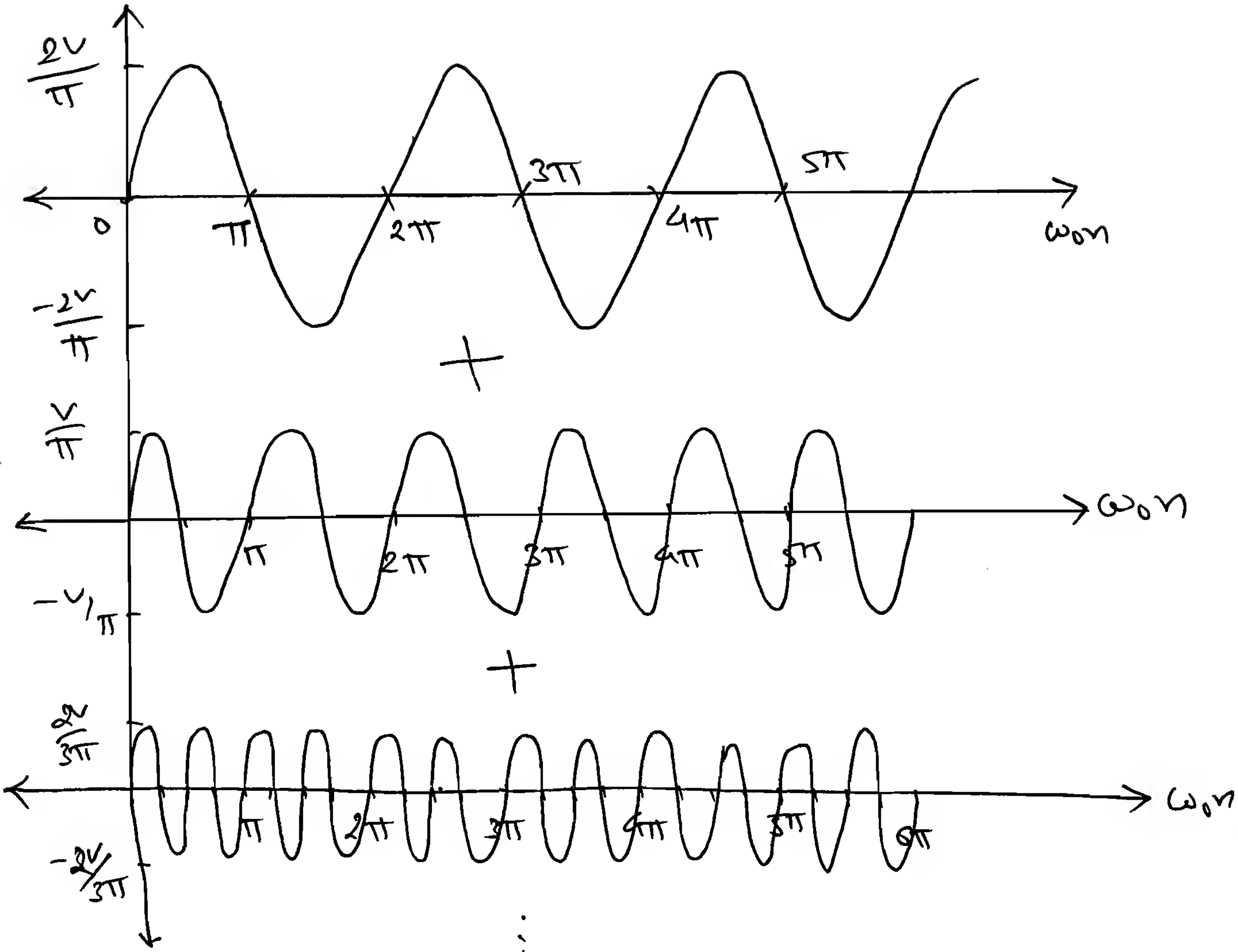
$$|d_n| = \sqrt{a_n^2 + b_n^2} = 0 + |b_n|.$$

$$\therefore |d_n| = |b_n| = \left| \frac{2V}{\pi n} \right|.$$

$$\therefore \quad \theta_n = \tan^{-1}(-b_n/a_n) = \frac{\pi}{2}, \quad n - \text{even}$$

$$= -\frac{\pi}{2}, \quad n - \text{odd}.$$





⋮

*		Phase
Real & Even	⟹	$0°$ & $\pm 180°$
Real & Odd	⟹	$\pm \pi/2$.

$\boxed{P\ 3.2.1.}$ A Periodic signal is given by $x(t) = 3\sin(4t + 30°) - 4\cos(12t - 60°)$ find the amplitude of second harmonic?

Soln:

$$x(t) = \underbrace{3\sin(4t + 30°)}_{\text{1st har.}} \underbrace{-4\cos(12t - 60°)}_{\text{3rd har.}}.$$

$\rightarrow$ $\omega_0 = G.C.D.\ (4, 12) = 4.$

So, Amp. II^{nd} har $= 0$.

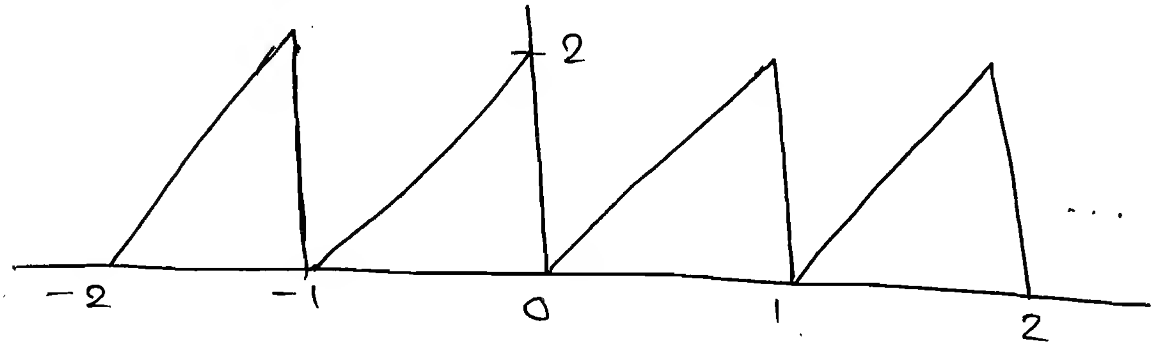
$\therefore$ phase of III^{rd} harmonic $= -60 \pm 180^\circ$

$1 + j0 \Rightarrow 0^\circ$

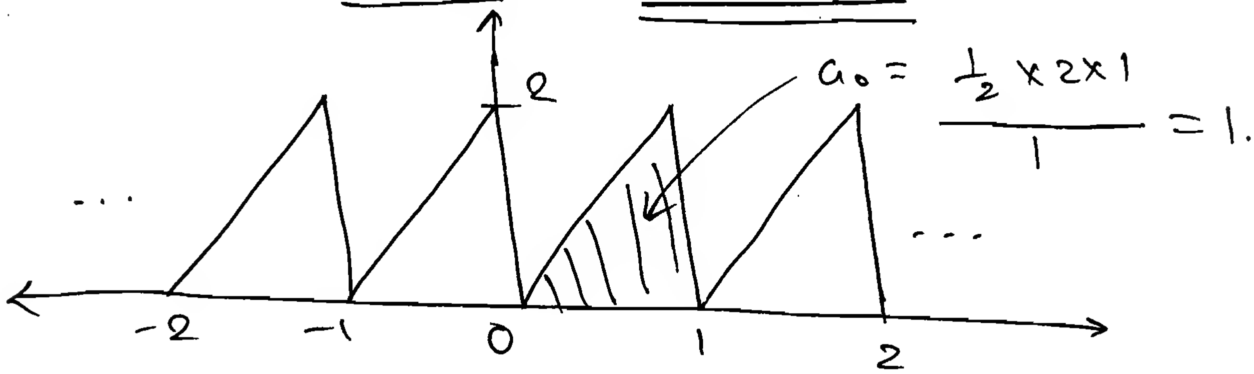
$-1 + j0 \Rightarrow \pm 180^\circ$

$+j \Rightarrow \pm 90^\circ$

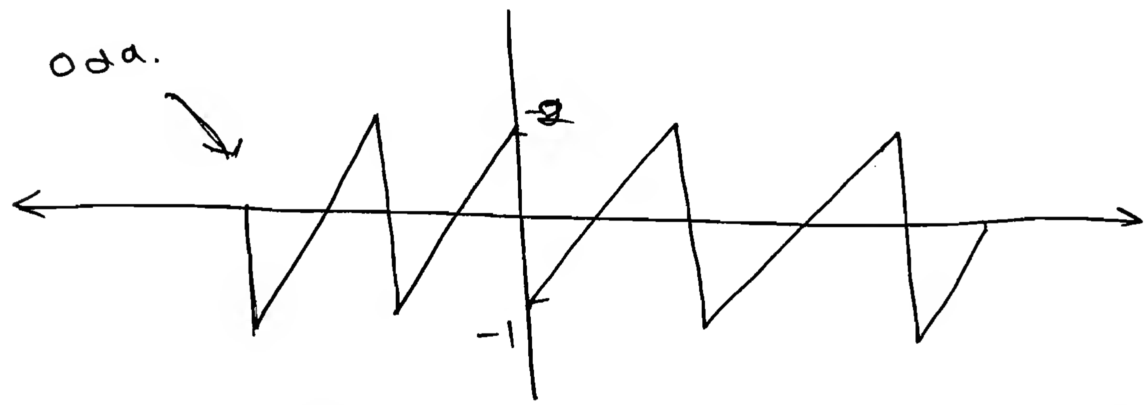
$\boxed{P\ 3.2.3}$ Find the T.F.S. representation of the periodic signal $x(t)$ shown in fig 3.2.37.



Soln: It is a hidden symmetry,



-1



$\Rightarrow$ for hidden odd symmetry we will get a_0 & b_n only.

$\boxed{\text{Hidden odd : } a_0 \text{ \& } b_n.}$

Here, $\boxed{a_0 = 1}$ $x(t) = 2t, \quad 0 \le t \le 1.$

$$b_n = \frac{2}{1}\int_0^1 2t \cdot \sin\omega_0 n \cdot dt$$

Here, $T = 1 \Rightarrow \omega_0 = 2\pi$

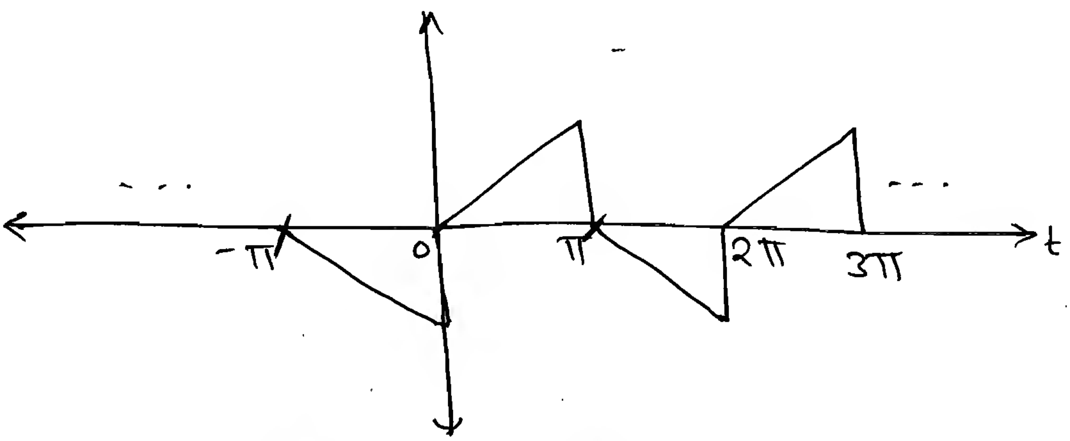
$$= \frac{2}{1} \times 2 \cdot \left[(t)\cdot\left(-\frac{\cos\omega_0 n}{\omega_0 n}\right) - (1)\left(-\frac{\sin\omega_0 n}{(\omega_0 n)^2}\right)\right]_0^1$$

$$= \cancel{4}^{2}\left[-\frac{1 \cdot \cos 2\pi n}{\cancel{2}\pi n} + 0 - 0 - 0 \right].$$

$$\boxed{b_n = \frac{-2}{\pi n}}$$

$\boxed{\text{P 3.2.4.}}$ For the Periodic waveforms show in figure what ~~odd~~ freq. Components are present in trignometric series expansion.

(P)

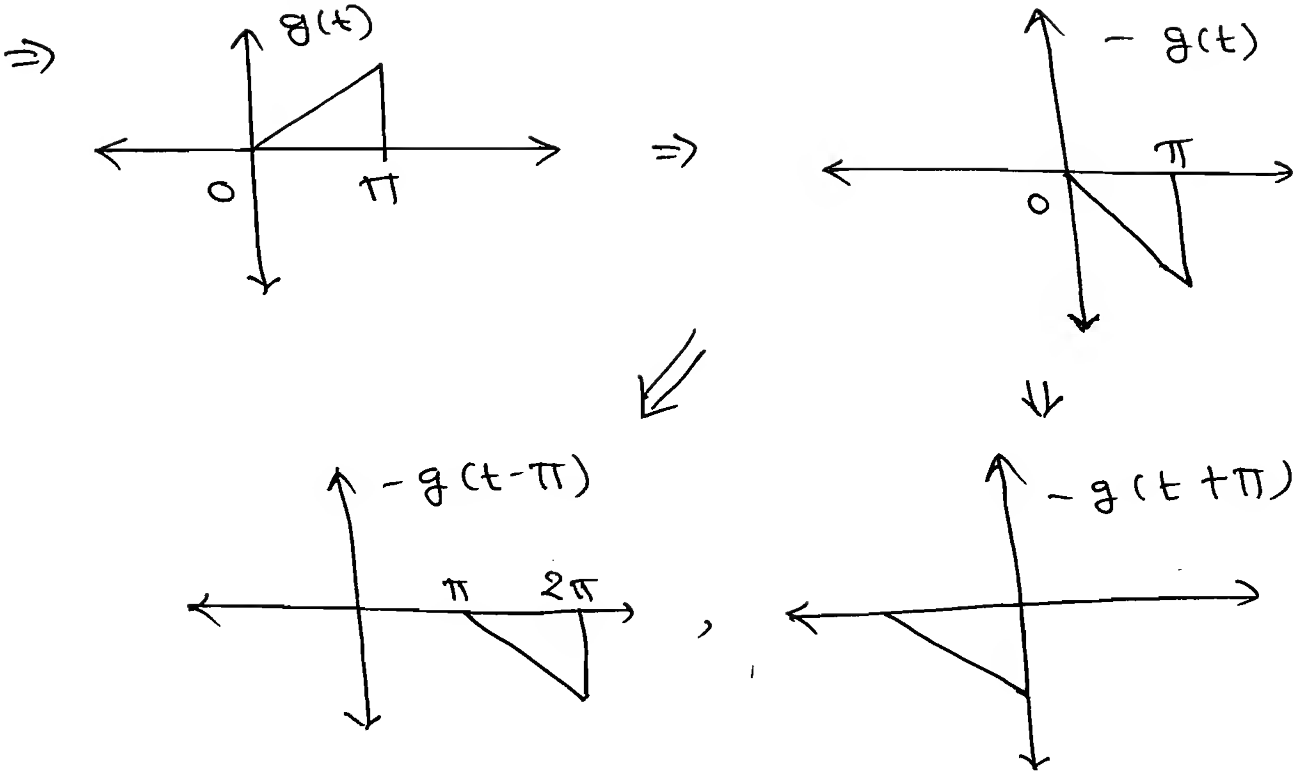


Sol^n:

$|\xleftarrow{\quad T = 2\pi \quad}|$

$\Rightarrow \omega_0 = \frac{2\pi}{2\pi} = 1$ rad/sec.

$\Rightarrow$

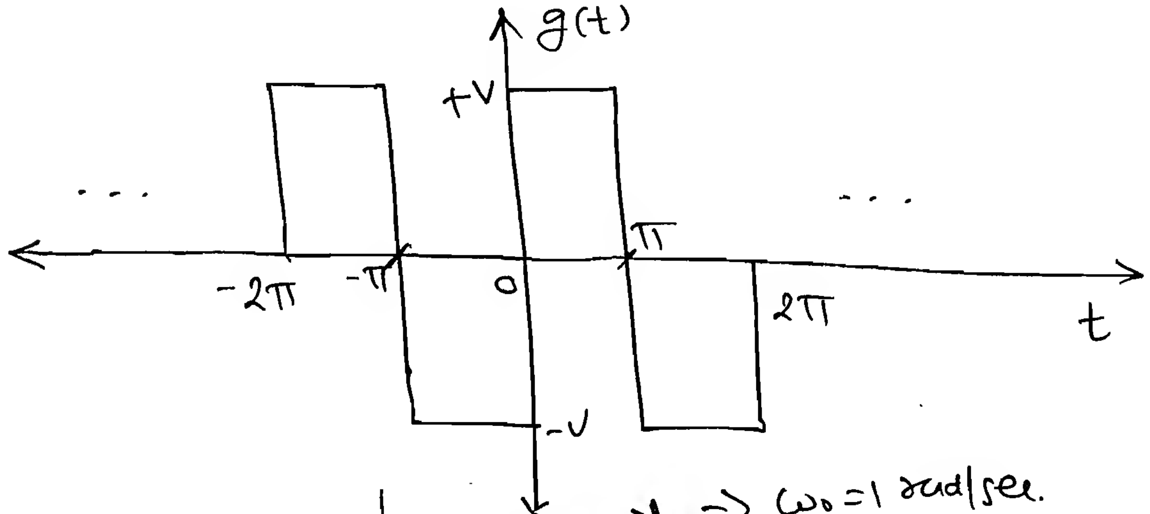


So, $g(t) = -g(t \pm T/2)$.

So, Half-wave symmetry.

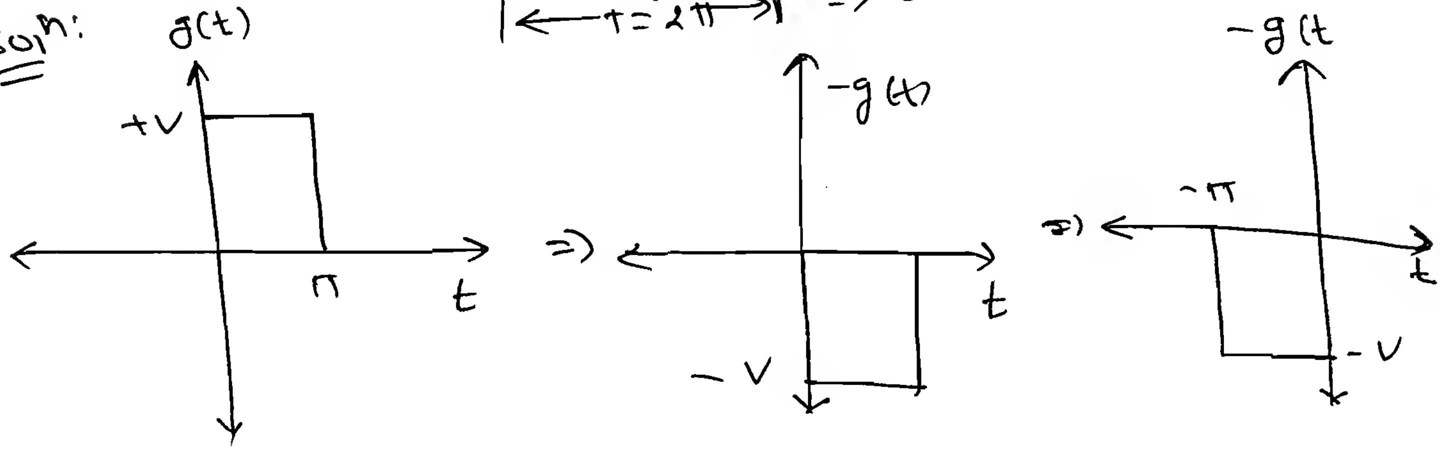
Hence, only odd harmonics.

[a]



$|\longleftarrow T = 2\pi \longrightarrow| \Rightarrow \omega_0 = 1$ rad/sec.

Soln:

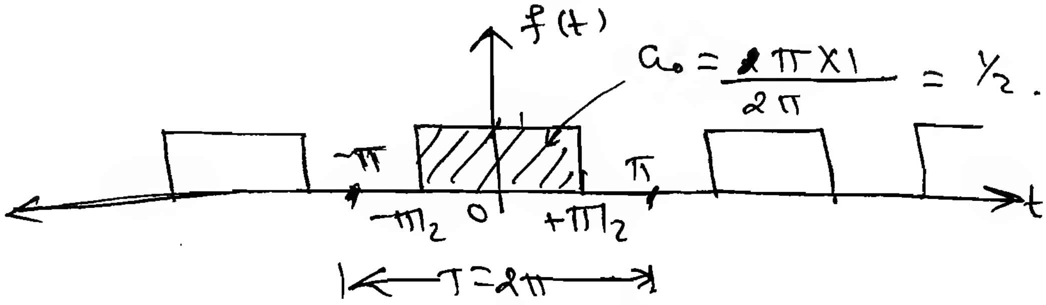


So, $g(t) = -g(-t)$ as well as $g(t) = -g(t \pm T/2)$.

→ Therefor given g(t) has odd and half-wave symmetry both.

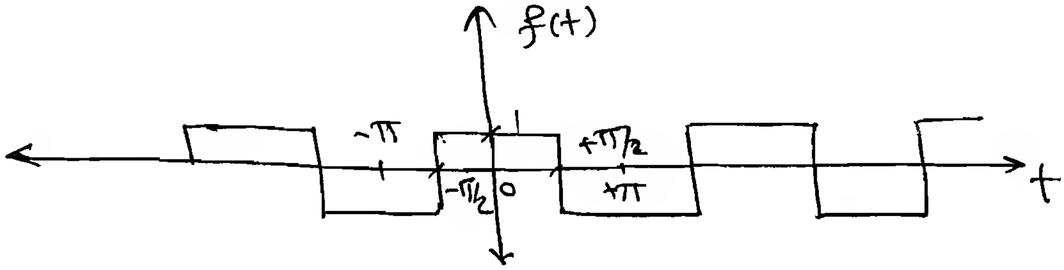
Hence, sine terms with odd harmonics.

(R)



$\omega_0 = 1$ rad/sec.

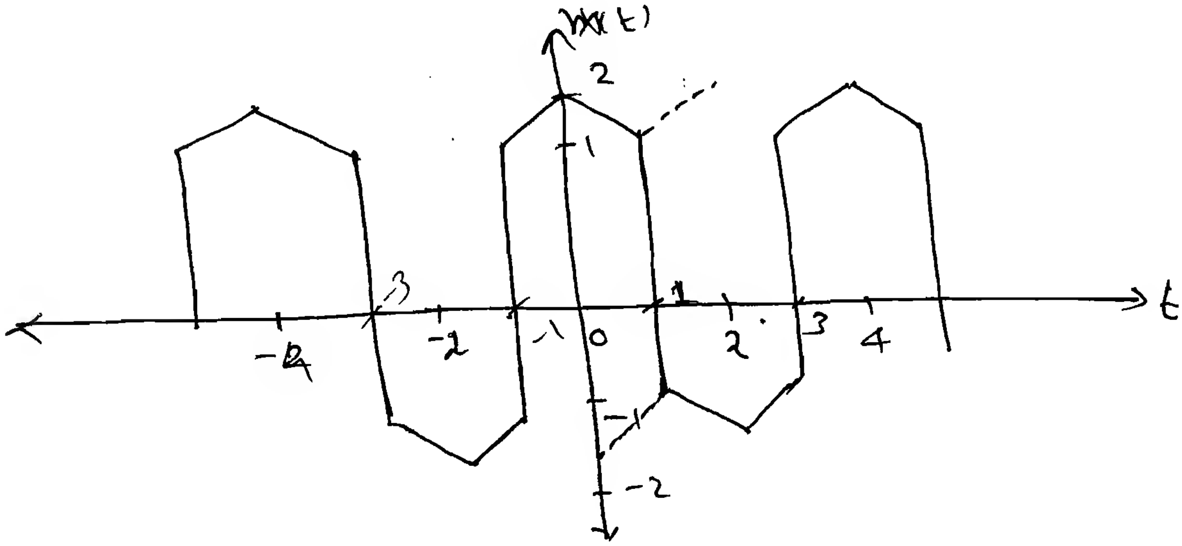
Sol^n:



So, It is a half-wave symmetry and also even symmetry.

So, DC, cosine terms with odd harmonics.

(S)



Sol^n: Here, given m(t) has even symmetry as well as half-wave symmetry. So, cosine terms with odd harmonics

$\boxed{P\ 3.2.5.}$ Consider the signal $x(t) = 10\cos(10\pi t + \pi/7) + 4\sin(30\pi t + \pi/8)$. It's power lying within the frequency band 10 Hz to 20 Hz is ____.

Sol^n:

$\omega_0 =$ G.C.D. $(10\pi, 30\pi)$

$\omega_0 = 10\pi$

$f = 5$ Hz.

$$x(t) = 10\cos(\underset{f = 5\,Hz}{10\pi t} + \pi/7) + 4\sin(\underset{15\,Hz}{30\pi t} + \pi/8)$$

So, power in the freq. band 10 Hz to 20 Hz is $\frac{4^2}{2} = 8W$.

$\boxed{Q - 3.2.6}$ Consider the trignometric series, which holds true $\forall t$, given by

$x(t) = \sin\omega_0 t + \frac{1}{3}\sin 3\omega_0 t + \frac{1}{5}\sin 5\omega_0 t + \frac{1}{7}\sin 7\omega_0 t + \cdots$

At $\omega_0 = \frac{\pi}{2}$ the series converges to ____.

Sol^n: at $\omega_0 = \pi/2$

$x(t) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

$= \pi/4.$

$\tan^{-1}x = x - \frac{x^2}{3} + \frac{x^3}{5} - \frac{x^4}{7} + \frac{x^5}{15} + \cdots$

$\therefore x = 1$

$\tan^{-1}(1) = \frac{\pi}{4} = 1 - \frac{1}{3} + 1/5 - 1/7 + \frac{1}{15} + \cdots$

P 3.2.7 A function is given by $f(t) = \sin^2 t + \cos 2t$. Which of the following is TRUE?

Soln:

$$f(t) = \sin^2 t + \cos 2t.$$

$$\Rightarrow f(t) = \frac{1 - \cos 2t}{2} + \cos 2t.$$

$$= \frac{1}{2} + \frac{\cos 2t}{2}.$$

$f = 0$ $\quad\quad$ $\omega_0 = 2 \Rightarrow f = 1/\pi$.

So, f has frequency components at 0 and $1/\pi$ Hz.

P 3.2.8. (a) The fundamental freq. of the composite signal.

$$x(t) = 2 + 3\cos(0.2t) + \cos(0.25t + \pi/2) + 4\cos(0.3t - \pi) \text{ is } \_\_.$$

Soln:

$$\omega_0 = \text{G.C.D.}(0.2, 0.25, 0.3).$$

$$= 0.05 \ (4, 5, 6).$$

So, $\boxed{\omega_0 = 0.05 \text{ rad/sec.}}$

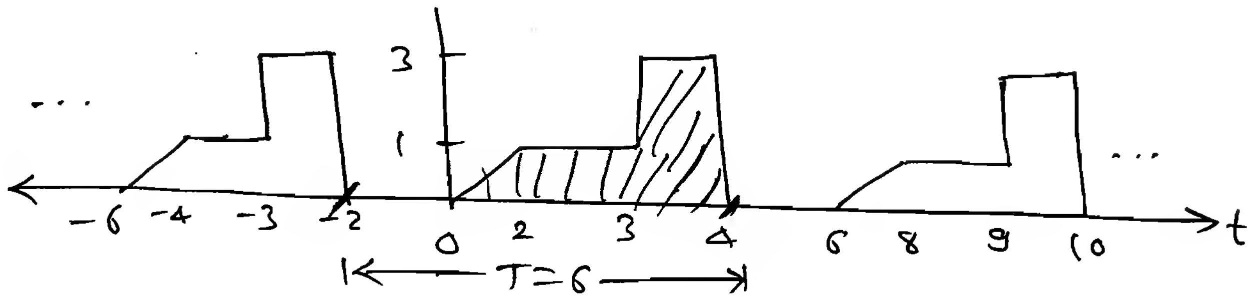
(b) For a periodic signal $v(t) = 30\sin 100t + 10\cos 300t + 6\sin(500t + \pi/4)$, the fundamental freq. in rad/s is.

Soln:

$$\omega_0 = \text{G.C.D.}(100, 300, 500).$$

$$\boxed{\omega_0 = 100 \text{ rad/s.}}$$

P 3.2.9. The average value of the periodic signal $x(t)$ shown in figure is,



Sol^n:

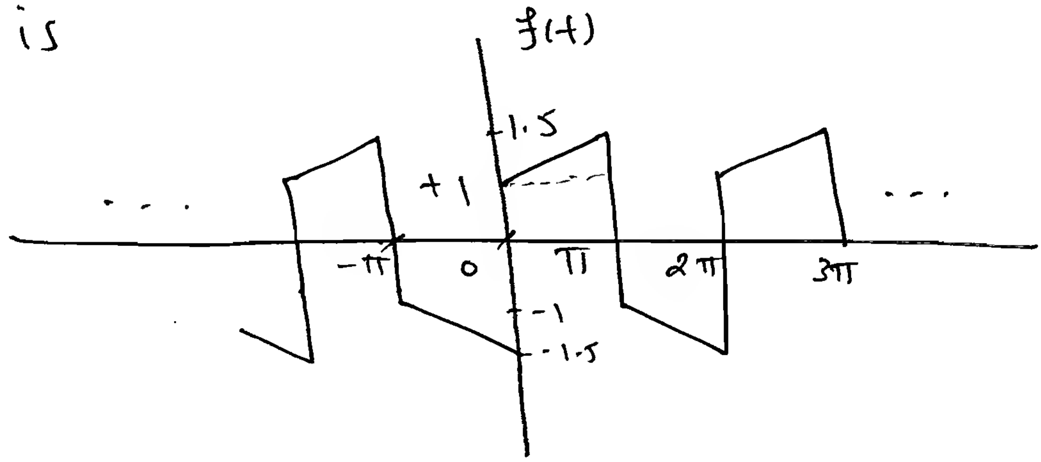
$$\text{avg. value} = \frac{\text{Area under curve over a period}}{\text{fundamental period.}}$$

$$\Rightarrow \text{avg. value} = \frac{(\frac{1}{\cancel{2}} \times \cancel{2} \times 1) + (1 \times 1) + (3 \times 1)}{6}$$

$$= 5/6.$$

P 3.2.10 $f(t)$, shown in fig. is represented by

$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt$. The value of a_0 is



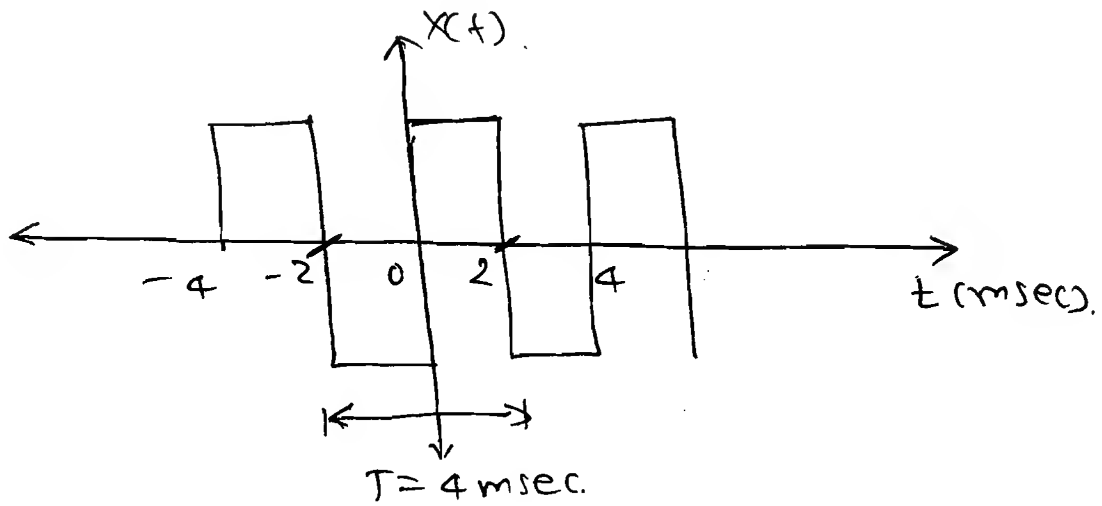
Sol^n:

$f(t)$ has odd symmetry hence,

$$\boxed{a_0 = 0.}$$

P 3.2-11 A periodic rectangular signal $x(t)$ has the waveform shown in fig. freq. of the fifth harmonic of its spectrum is

⇒



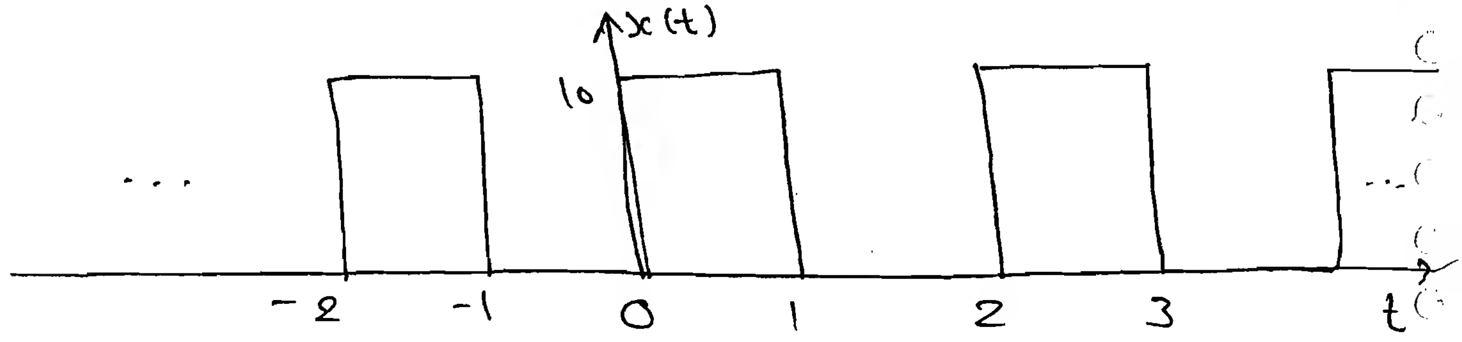
$\therefore \; f_0 = 1/T = \frac{1000}{4} = 250\,Hz$

$\therefore$ freq. of the fifth harmonic is

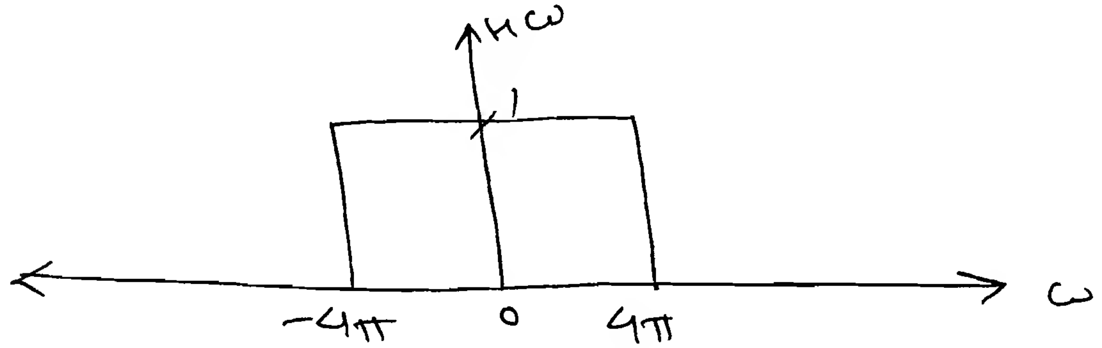
$f_5 = 5 \times f_0 = 1250\,Hz.$

$\boxed{P\,3.2.12}$ A Periodic input signal x(t) Shown below is applied to an L.T.I. system with freq. response.

$$H(\omega) = \begin{cases} 1; & |\omega| < 4\pi \\ 0; & |\omega| > 4\pi \end{cases}$$ find the o/p?



Sol:



$\Rightarrow$ we should find $x(t)$ in frer. domain.

$$T = 2 \Rightarrow \omega_0 = \frac{2\pi}{2} = \pi$$

$$a_0 = \frac{10 \times 1}{2} = 5.$$

$$a_n = \frac{2}{T} \int_0^T x(t) . \cos \omega_0 n . t \, dt$$

$$= \frac{2}{2} \int_0^2 \cancel{x}(t) . \cos \omega_0 n . t . \, dt.$$

$$= \int_0^1 10 . \cos \omega_0 n t . \, dt .$$

$$= 10 \int_0^1 . \cos \pi n t . \, dt$$

$$= 10 . \left[\frac{\sin \pi n t}{\pi n} \right]_0^1 = 0.$$

$$b_n = \frac{2}{2} \int_0^1 10 . \sin \omega_0 n t \, dt .$$

$$= 10 . \left[- \frac{\cos \pi n t}{\pi n} \right]_0^1$$

$$= 10 \left[\frac{1 - \cos \pi n}{\pi n} \right].$$

$$b_n = \frac{10}{\pi n} \left[1 - (-1)^n \right].$$

$$\therefore b_n = \frac{20}{\pi n} , \quad \text{'n-odd'}$$

$$b_n = 0 , \quad \text{'n-even'}.$$

$$x(t) = a_0 + \sum_{n \geq 1}^{\infty} a_n \cos \omega_0 n t + b_n \sin \omega_0 n t.$$

$$\therefore \quad x(t) = 5 + \sum_{n=1}^{\infty} \frac{10}{\pi n}\left[1-(-1)^n\right] . \sin\omega_0 n.$$

$$\therefore \quad x(t) = 5 + \frac{20}{\pi}\cos\omega_0 n + \frac{20}{3\pi}\cos 3\omega_0 n + \frac{20}{5\pi}\cos 5\omega_0 n + \cdots$$

$\omega_0 = \pi$

$$x(t) = 5 + \frac{20}{\pi}\cos\pi n + \frac{20}{3\pi}\cos 3\pi n + \frac{20}{5\pi}\cos 5\pi n + \cdots$$

So, first 3-terms passed through given filter.

* Exponential Fourier Series (or) Complex Fourier series:

$\Rightarrow$

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\omega_0 nt + b_n \sin\omega nt.$$

$$= a_0 + \sum_{n=1}^{\infty} a_n\left[\frac{e^{j\omega_0 nt} + e^{-j\omega_0 nt}}{2}\right] + b_n\left[\frac{e^{j\omega_0 nt} - e^{-j\omega_0 nt}}{2j}\right]$$

$$= \underbrace{a_0}_{c_0} + \sum_{n=1}^{\infty} e^{j\omega_0 nt}\underbrace{\left[\frac{a_n - jb_n}{2}\right]}_{c_n} + e^{-j\omega_0 nt}\underbrace{\left[\frac{a_n + jb_n}{2}\right]}_{c_{-n}}.$$

$$\therefore\ g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot e^{j\omega_0 n t} + \sum_{n=1}^{\infty} C_{-n} \cdot e^{-j\omega_0 n t}.$$

Put $n=-m$

$$\sum_{m=-1}^{-\infty} C_m \cdot e^{+j\omega_0 m t}.$$

$$= \sum_{n=-1}^{-\infty} C_n \cdot e^{j\omega_0 n t}.$$

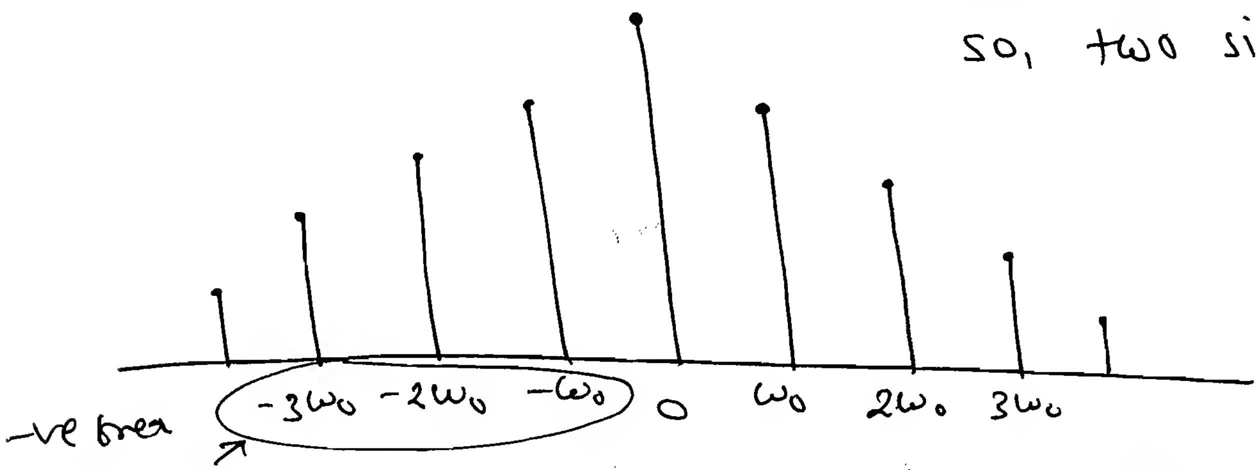
$$\therefore\ g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot e^{j\omega_0 n t} + \sum_{n=-\infty}^{-1} C_n \cdot e^{+j\omega_0 n t}.$$

$$\therefore\ \boxed{g(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{j\omega_0 n t}.}$$

$\Rightarrow$ +ve and -ve freqs. denotes opposite direction of rotation i.e. opposite phase angle.

$\Rightarrow -\infty$ to $+\infty$

so, two sided.



Consider because Reproducing the original signal

☆ T.F.S. to E.F.S. :

$$\Rightarrow \boxed{C_0 = a_0}$$

$$C_n = \frac{a_n - jb_n}{2}$$

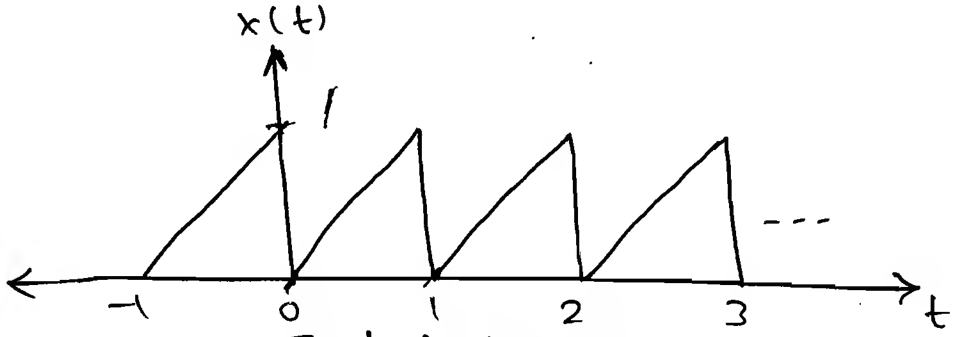
$$\rightarrow \boxed{C_n = \frac{1}{T}\int_0^T g(t) \cdot e^{-jn\omega_0 t} \cdot dt}$$

$$\rightarrow \boxed{C_{-n} = \frac{a_n + jb_n}{2} = \frac{1}{T}\int_0^T g(t) \cdot e^{j\omega_0 n t} \cdot dt}$$

☆ E.F.S. to T.F.S.

$$\Rightarrow \boxed{\begin{aligned} a_0 &= C_0 \\ a_n &= C_n + C_{-n}. \\ b_n &= C_{-n} - C_n. \end{aligned}}$$

P 3.2.15 obtain the E.F.S. representation of periodic signal shown, hence find the T.F.S. ?



$T = 1 \Rightarrow \omega_0 = 2\pi$

Soln: $x(t) = t, \quad 0 < t < 1.$

$\boxed{\omega_0 = 2\pi}$

$$c_n = \frac{1}{T}\int_0^T x(t) \cdot e^{-j\omega_0 n t} \cdot dt.$$

$$\therefore c_n = \frac{1}{1}\int_0^1 t \cdot e^{-j2\pi n t} \cdot dt.$$

$$c_n = \left[(t)\left(\frac{e^{-j2\pi n t}}{-j2\pi n} \right) - (1)\left(\frac{e^{-j2\pi n t}}{(2\pi n)^2} \right) \right]_0^1$$

$$c_n = \frac{(1) \cdot e^{-j2\pi n}}{-j2\pi n} - \frac{e^{-j2\pi n}}{(2\pi n)^2} + \frac{e^0}{(2\pi n)^2}.$$

$$= \frac{j}{2\pi n} - \cancel{\frac{1}{2\pi n^2}} + \cancel{\frac{1}{(2\pi n)^2}}$$

$$\boxed{c_n = \frac{j}{2\pi n}}, \quad \boxed{c_{-n} = \frac{-j}{2\pi n}}$$

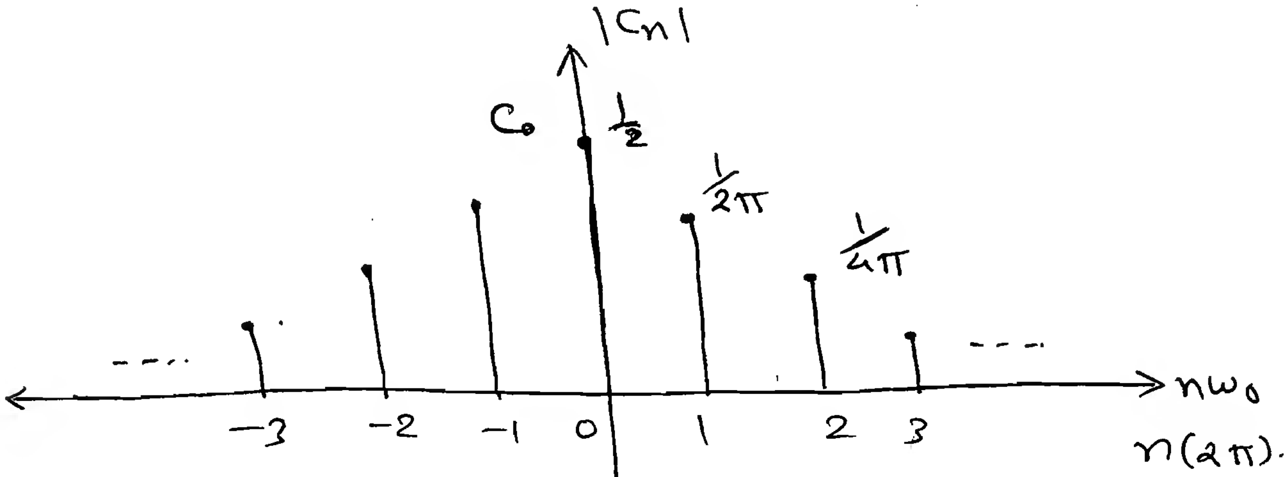
$$\Rightarrow c_0 = \tfrac{1}{2} \times (1) \times (1) = \frac{1}{2}.$$

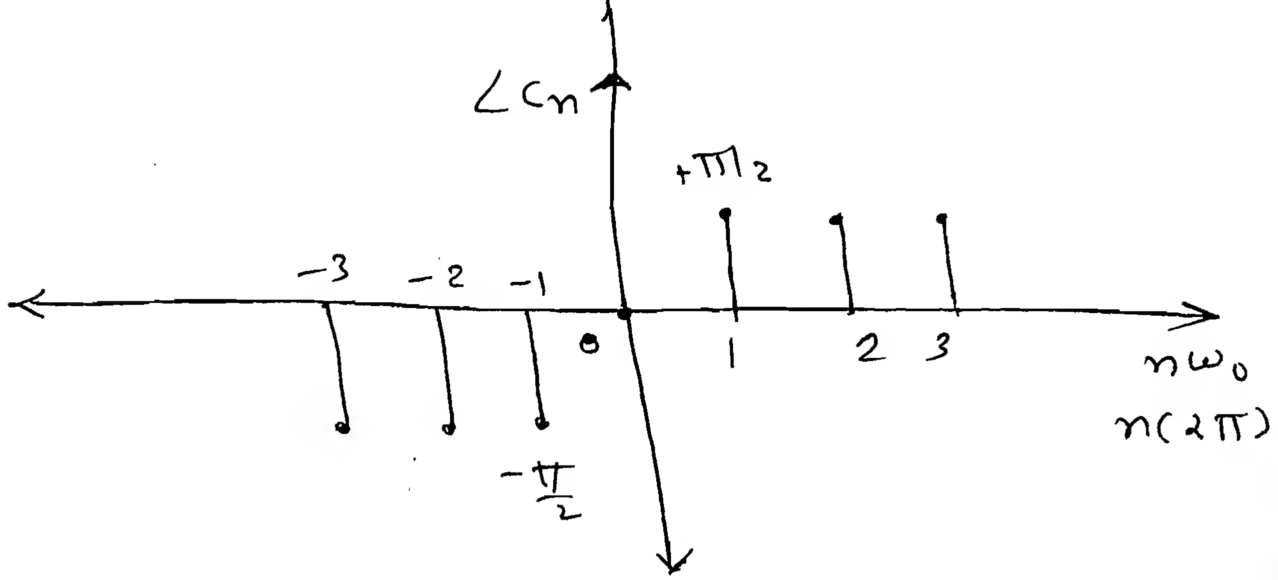
$$\boxed{c_0 = \tfrac{1}{2}}$$

$$\angle c_n = \frac{\pi}{2}, \quad n \to +ve.$$
$$= -\frac{\pi}{2}, \quad n \to -ve.$$

$$\Rightarrow |c_n| = \left| \frac{j}{2\pi n} \right|$$

$$|c_n| = \frac{1}{2\pi n}, \qquad |c_0| = \tfrac{1}{2}.$$





$$\Rightarrow \quad a_n = \frac{C_n + C_{-n}}{2}$$

$$a_n = \frac{\frac{j}{2\pi n} - \frac{j}{2\pi n}}{2} = 0$$

$$\boxed{a_n = 0}$$

$$\Rightarrow \quad j\, b_n = \frac{C_{-n} - C_n}{\cancel{2}\ 1} = \frac{-\frac{j}{2\pi n} - \frac{j}{2\pi n}}{\cancel{2}\ 1}$$

$$\boxed{b_n = \frac{1}{\pi n}} \qquad a_0 = C_0$$

$$\boxed{a_0 = 1/2}$$

$$\Rightarrow \quad |d_n| = \sqrt{a_n^2 + b_n^2}$$

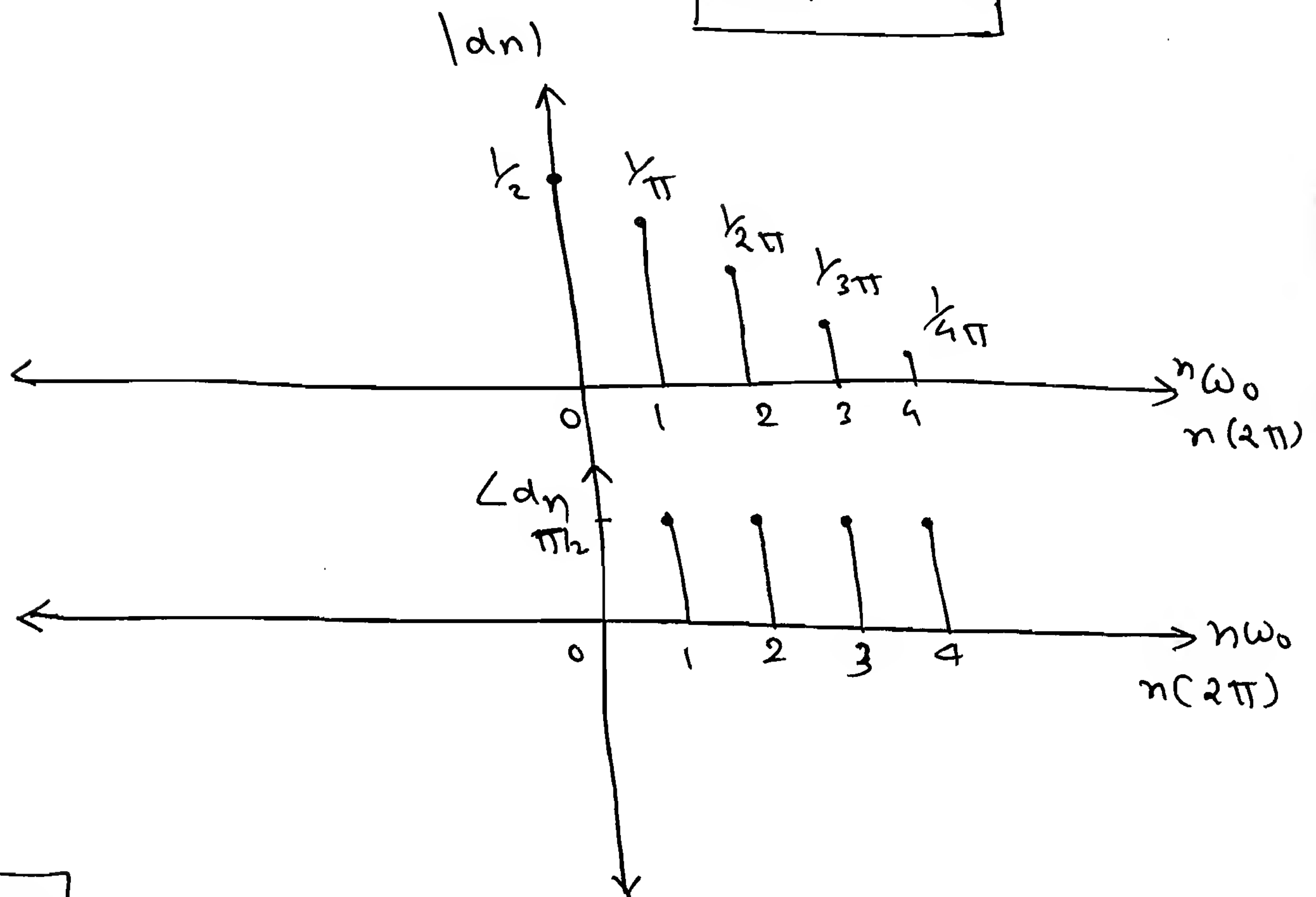
$$|d_n| = \sqrt{0 + \left(\frac{1}{\pi n}\right)^2}$$

$$\therefore \boxed{|d_n| = \frac{1}{\pi n}}$$

$$\angle d_n = +\tan^{-1}\left(-b_n/a_n\right).$$

$$\angle d_n = \tan^{-1}\left(+\frac{1}{\pi n}/0\right).$$

$$\boxed{\angle d_n = \pi/2}$$



P3.2.14

a For the periodic signal

$$x(t) = 2 + \cos\left(\frac{2\pi t}{3}\right) + 4\sin\left[\frac{5\pi t}{3}\right],$$ find the E.F.S. coefficients?

Soln:

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{+j\omega_0 n t}.$$

$$= 2 + \frac{e^{j(2)(\frac{\pi}{3})t} + e^{-j(2)(\frac{\pi}{3})t}}{2} + 4\left[\frac{e^{j5(\frac{\pi}{3})t} - e^{-j5(\frac{\pi}{3})t}}{2j}\right]$$

So, $\boxed{C_0 = 2}$ $\boxed{C_2 = C_{-2} = \frac{1}{2}}$

$C_5 = \frac{4}{2j}$ $\boxed{C_{-5} = +2j}$

$\boxed{C_5 = -2j}$

$\boxed{Q}$ The F.S. Representation of periodic signal is $x(t) = \sum_{n=-\infty}^{+\infty} \frac{3}{4+(n\pi)^2} \cdot e^{jn\pi t}$ if one of the component is $a\cos 3\pi t$, Find a.

Soln: here, $\boxed{\omega_0 = \pi}$

$A\cos 3\pi t \longrightarrow$ 3rd harmonic.

$\therefore \quad a_n = C_n + C_{-n}$.

$\therefore \quad = C_3 + C_{-3}$.

$\Rightarrow \quad C_n = \frac{3}{4+(n\pi)^2}$

$C_3 = \frac{3}{4+(3\pi)^2}$

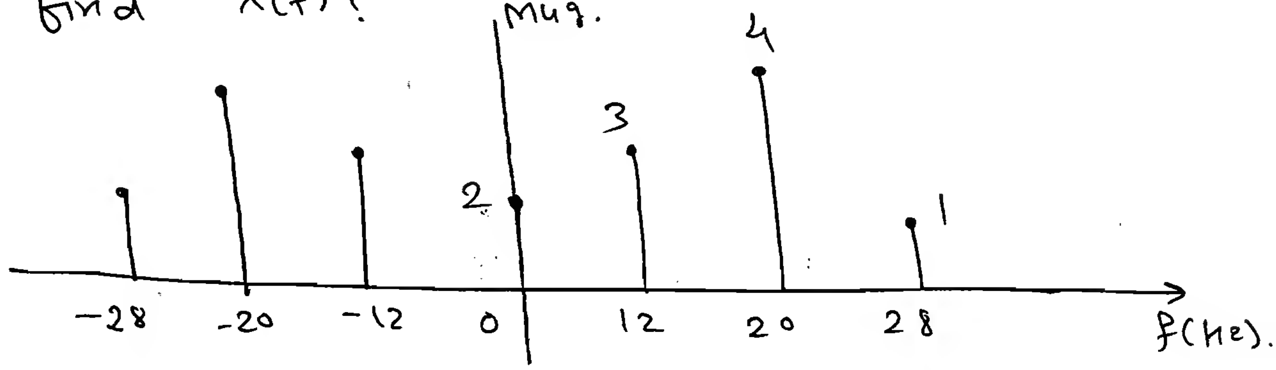
$C_{-3} = \frac{3}{4+9\pi^2}$.

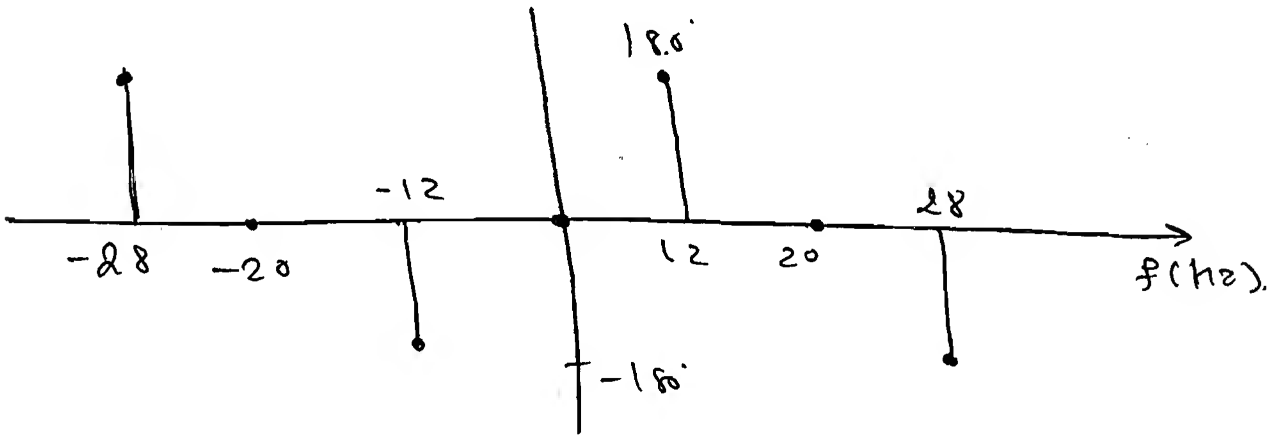
$\therefore \quad a_n = \frac{3}{4+9\pi^2} + \frac{3}{4+9\pi^2}$.

$$\boxed{a_n = \frac{6}{4+9\pi^2}}$$

Note: Cos fn corresponds to a_n.
sin fn corresponds to b_n.

$\boxed{P\ 3.2.16}$ Consider the two-sided signal spectrum shown in figure for signal $x(t)$, find $x(t)$?





Soln:

$f_0 =$ GCD (12, 20, 28)

$\boxed{f_0 = 4\ Hz}$

$\Rightarrow$ Polar form:

$$x(t) = d_0 + \sum_{n=-\infty}^{+\infty} d_n \cos(\omega_0 n t + \theta_n).$$

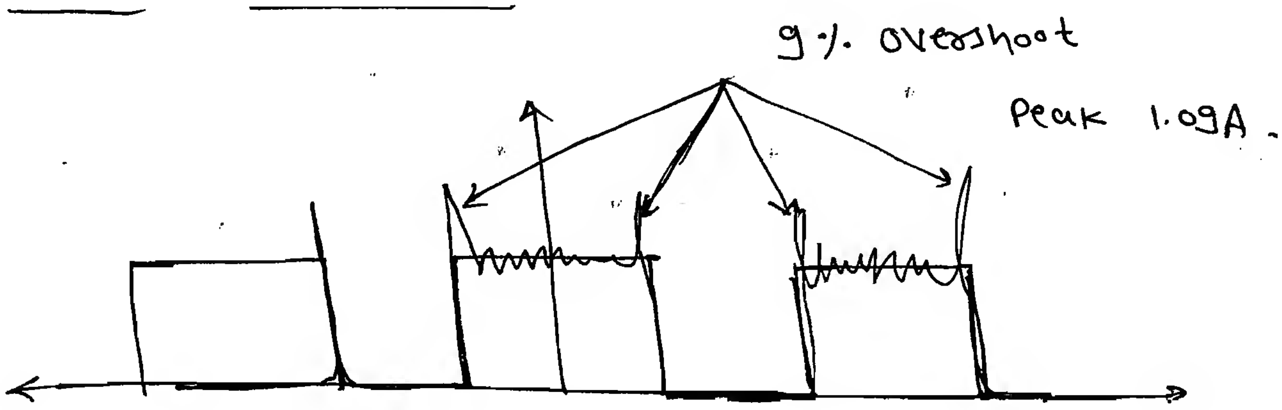
$$\boxed{\begin{aligned} x(t) &= 2 + 6\cos(2\pi(12)t + 180^\circ) \\ &+ 8\cos(2\pi(20)t + 0^\circ). \\ &+ 2\cos(2\pi(28) - 180^\circ). \end{aligned}}$$

* Convergence of FS:

⇒ $\left.\begin{matrix} a_0, a_n, b_n \\ c_0, c_n \end{matrix}\right\} < \infty \Rightarrow$ finite

⇒ Gibbs phenomena:

⇒



⇒ Irrespective of the no. of harmonics we are adding always at the point of discontinuity 9% overshoot is observed i.e. gibbs phenomena. (Truncation in time corresponds to ripples in freq. domain)

⇒ Dirchlet Condition:

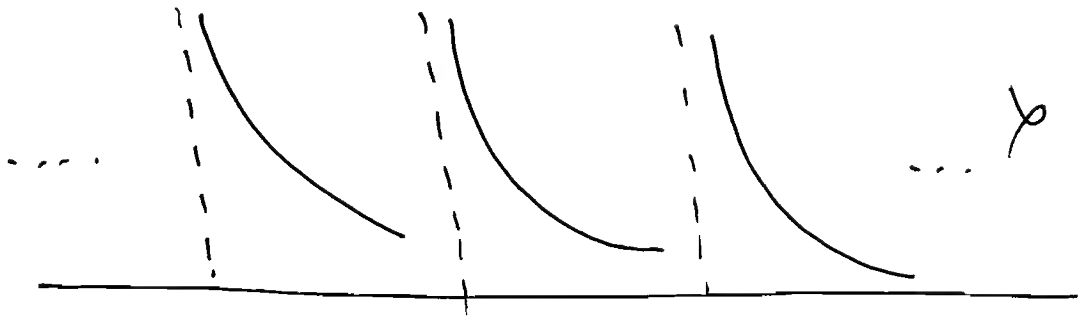
(1) x(t) is absolutely integrable.

i.e. $\int_0^T |x(t)|\,dt < \infty$.

(2) x(t) has only finite number of maxima and minima.

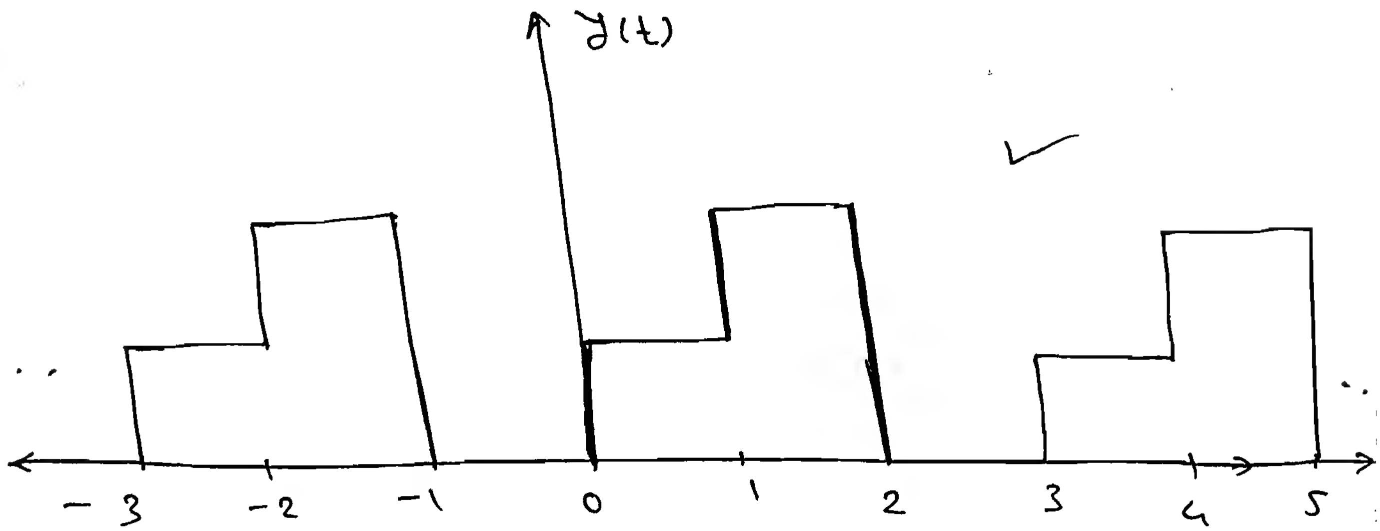
(3) The number of discontinuities in x(t) must be finite.

e.g. ① $x(t) = \frac{1}{t}, \quad 0 < t < 1.$



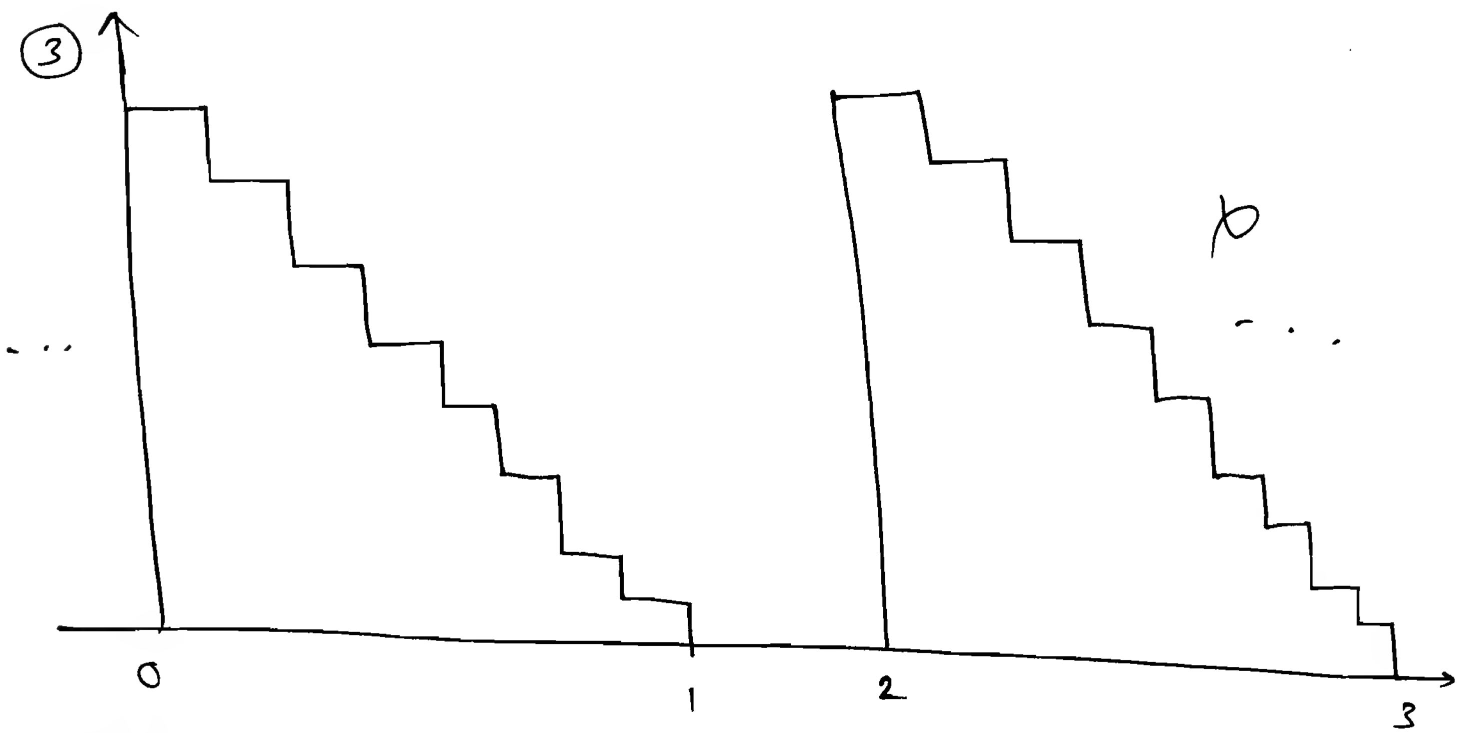
Not absolutely Integrable.

②



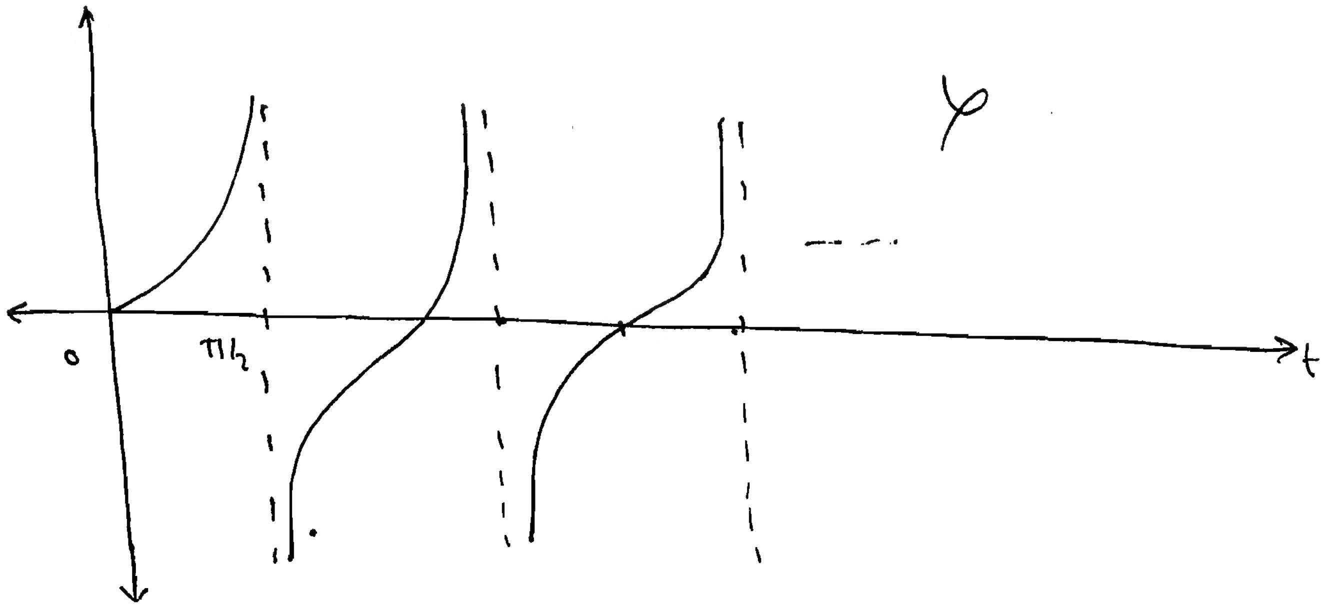
$\Rightarrow$ Valid because finite$^{(3)}$ discontinuity (3 to 4)

③



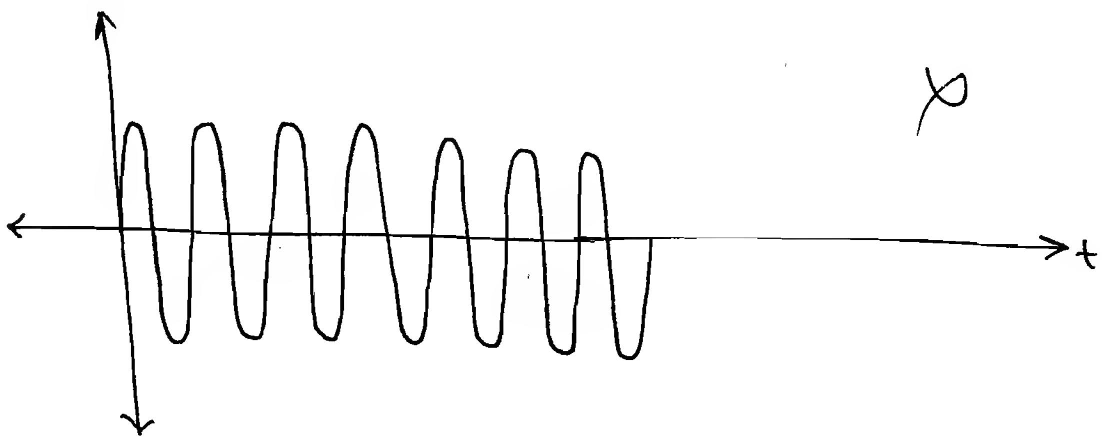
Not Valid because so many discontinuity.

④ $x(t) = \tan t$, $0 < t < \pi/2$.



Not valid because within the limit it has infinite discontinuity.

⑤ $x(t) = \sin\left(\frac{2\pi}{t}\right)$, $0 < t < 1$.



⇒ Not valid more no. of maxima & minima.

* Properties of F.S :-

(1) Linearity:

$\Rightarrow$ If $x_1(t) \longrightarrow C_n$

$x_2(t) \longrightarrow d_n$

then $\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha C_n + \beta d_n$.

(2) Time Shift:

$\Rightarrow$ If $x(t) \longrightarrow C_n$

the $x(t-t_0) \longrightarrow e^{-j\omega_0 n t_0} \cdot C_n$.

→ when we shift in the time-domain, it changes the phase of each harmonic in proportion to its freq. $n\omega_0$.

(3) Frequency Shift :-

$\Rightarrow$ If $x(t) \longrightarrow C_n$

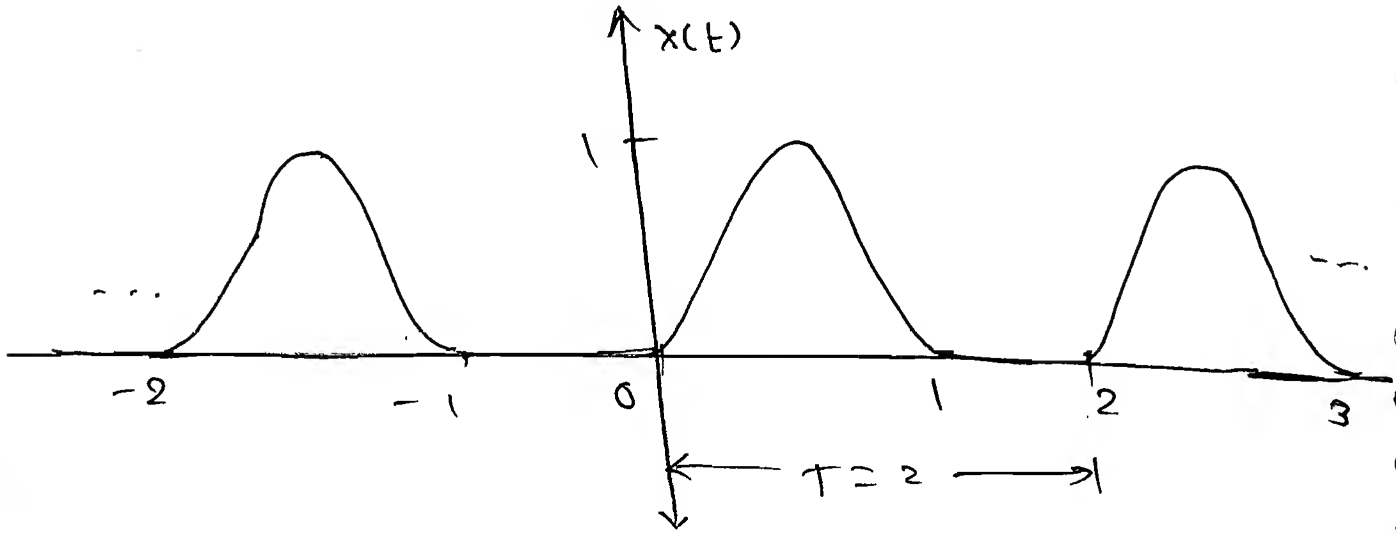
the $e^{j\omega_0 M t} \cdot x(t) \longrightarrow C_{n-M}$.

Note: Shifting one domain corresponds to multiplication by exponential term in other domain.

P 3.3.1. The F.S. coefficient of signal $x(t)$ shown in fig(a) are $C_0 = 1/\pi$, $C_1 = -j0.25$,

$$C_n = \frac{1}{\pi(1-n^2)} \quad (n \text{ even})$$

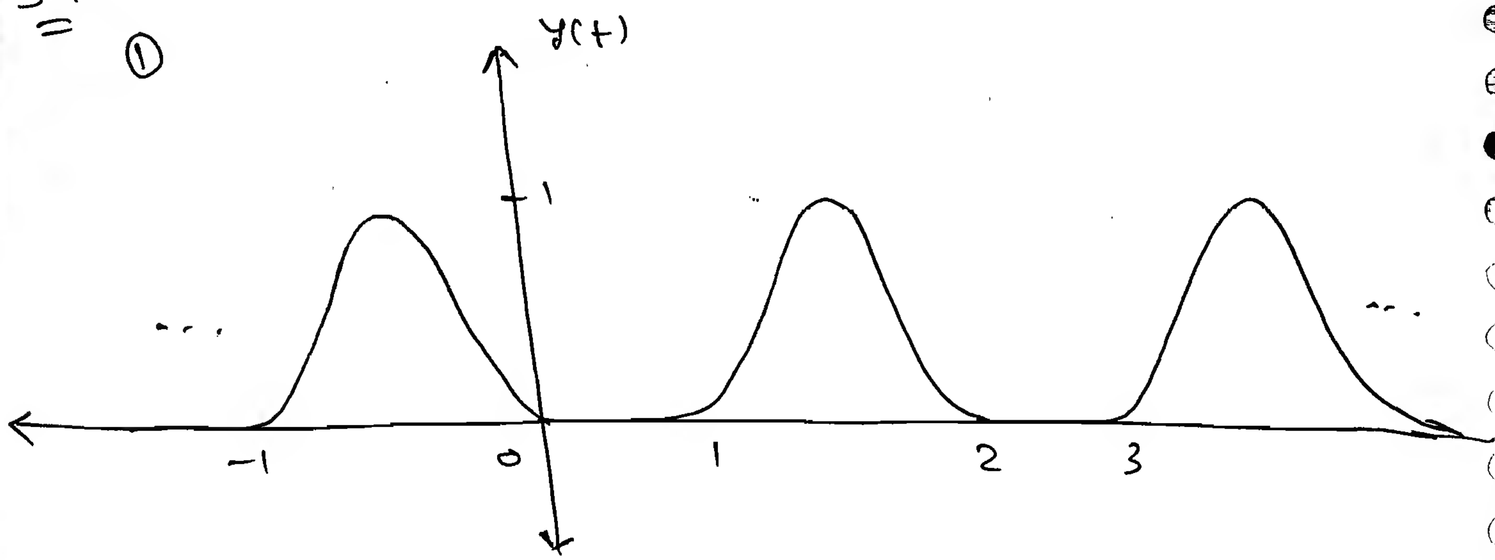
Find F.S. coefficient of $y(t)$, $f(t)$ and $g(t)$?



$$\Rightarrow \omega_0 = \frac{2\pi}{T} = \pi$$

Soln:

(1)



$\Rightarrow y(t) = x(t-1)$.

$x(t) \rightarrow c_n$.

$\therefore$

$\Rightarrow x(t-1) = y(t) \longrightarrow e^{-j\omega_0(1)\cdot n} \cdot C_n = d_n$

$\longrightarrow e^{-j\pi n} \cdot C_n = d_n$

$\therefore \quad d_n = (-1)^n . c_n.$

$\therefore \quad d_0 = (-1)^0 \cdot c_0 = c_0$

$$\boxed{d_0 = 1/\pi}$$

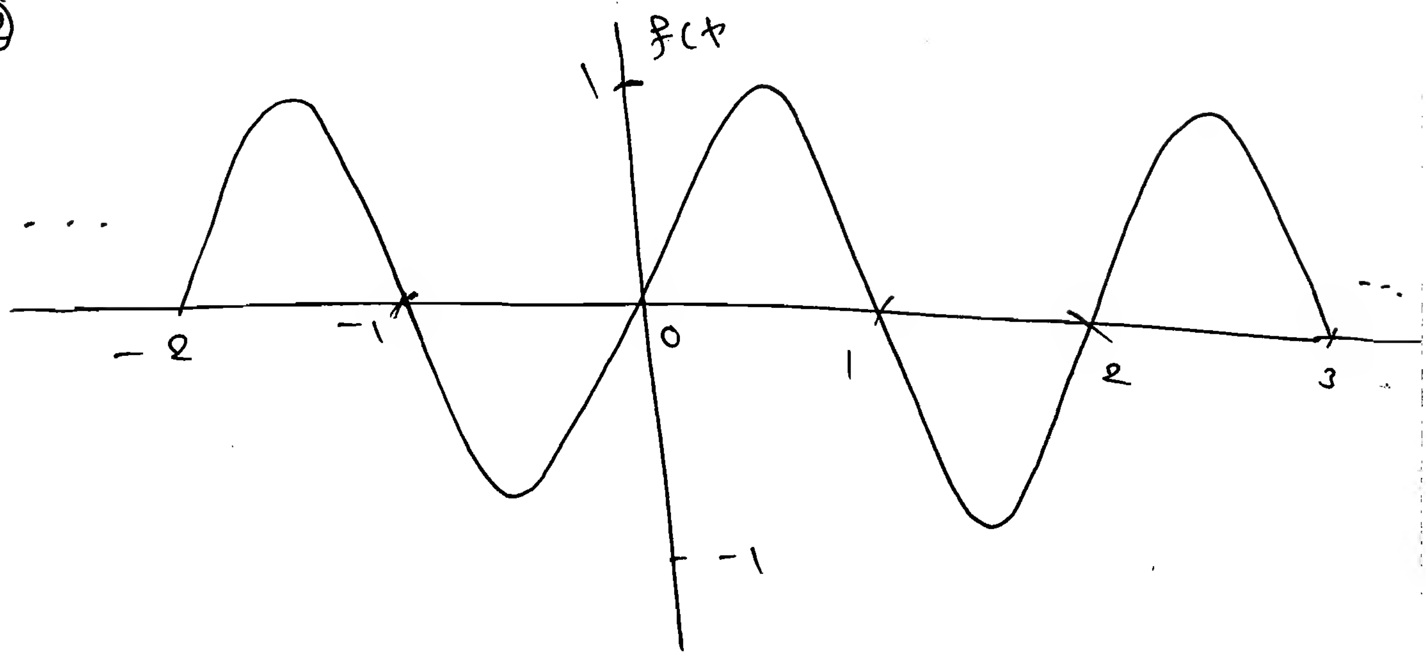
$\therefore \quad d_1 = (-1)^1 . c_1 = +j0.25$

$$\boxed{d_1 = +j0.25}$$

$\therefore \quad d_n = \dfrac{(-1)^n \times \pi}{(1-n^2)} \quad (n \text{ even}).$

$\therefore \quad \boxed{d_n = c_n. \quad (n \text{ even})}$

(2)



$\Rightarrow \quad 4 \quad f(t) = x(t) - x(t-1).$

$f(x) = x(t) - y(t).$

$\therefore \quad \downarrow$ linearity

$f_n = c_n - d_n.$

$\therefore \quad f_0 = c_0 - d_0 = 1/\pi - 1/\pi = 0. \Rightarrow \boxed{f_0 = 0}$

$\therefore \quad F_1 = c_1 - d_1$

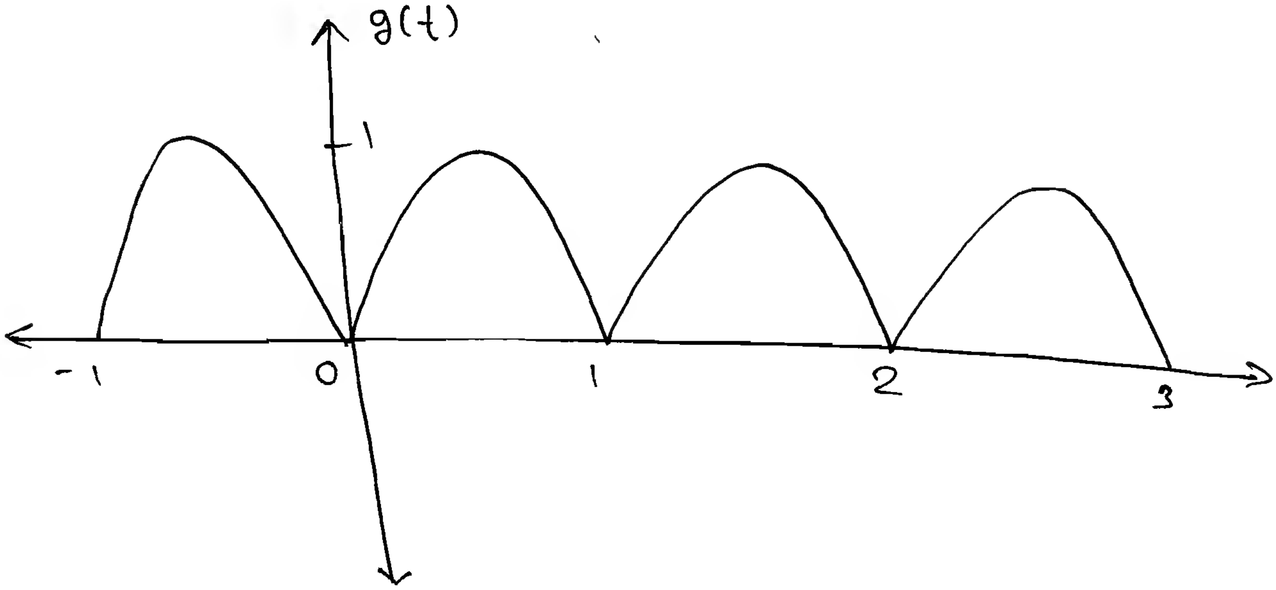
$= -j0.25 - j0.25$

$\boxed{F_1 = -j0.5}$

$\Rightarrow \quad F_n = c_n - d_n.$

$\boxed{F_n = 0}$.

③



$\Rightarrow \quad g(t) = x(t) + x(t-1) = x(t) + y(t)$

$\Downarrow$

$G_n = c_n + d_n.$

$\therefore \quad g_0 = c_0 + d_0 = 2/\pi$

$\boxed{g_0 = 2/\pi}$

$\Rightarrow \quad g_1 = c_1 + d_1$

$\boxed{g_1 = 0}$

$\Rightarrow \quad g_n = c_n + d_n$

$\boxed{g_n = \frac{2}{\pi(1-n^2)}}$ (n -even).

$\boxed{P\ 3.3.2}$ Let $x(t)$ be a periodic signal with period T and F.S. coefficient C_n.

Let, $y(t) = x(t-t_0) + x(t+t_0)$. The F.S. coefficient of $y(t)$

If $d_n = 0 \ \forall$ odd n then t_0 can be

a) $T/8$ b) $T/4$ c) $T/2$ d) $2T$.

Soln:

$$y(t) = x(t-t_0) + x(t+t_0).$$

$$\Downarrow$$

$$\therefore \ d_n = e^{-j\omega_0 n t_0} \cdot C_n + e^{j\omega_0 n t_0} \cdot C_n.$$

$$d_n = 2C_n \left[\frac{e^{j\omega_0 n t_0} + e^{-j\omega_0 n t_0}}{2} \right].$$

$$\therefore d_n = 2C_n \cos \omega_0 n t_0.$$

$\Rightarrow$ $d_n = 0$ when $\omega_0 n t_0 = \frac{n\pi}{2}$, n odd.

$$\therefore \ \omega_0 \cdot t_0 = \frac{\pi}{2}.$$

$$\therefore \ \frac{2\pi}{T} \cdot t_0 = \frac{\pi}{2}$$

$$\therefore \ \boxed{t_0 = T/4}$$

4) Time - Scaling:

$\Rightarrow$ $x(t) \longrightarrow C_n |_{T, \omega_0}$

$x(\alpha t) \longrightarrow C_n | _{T/\alpha, \alpha\omega_0}$.

Time - Compressing by α changes frequency from ω_0 to $\alpha\omega_0$.

$$\rightarrow \quad x(t) = \sum_{n=-\infty}^{+\infty} C_n \cdot e^{-j\omega_0 n t}.$$

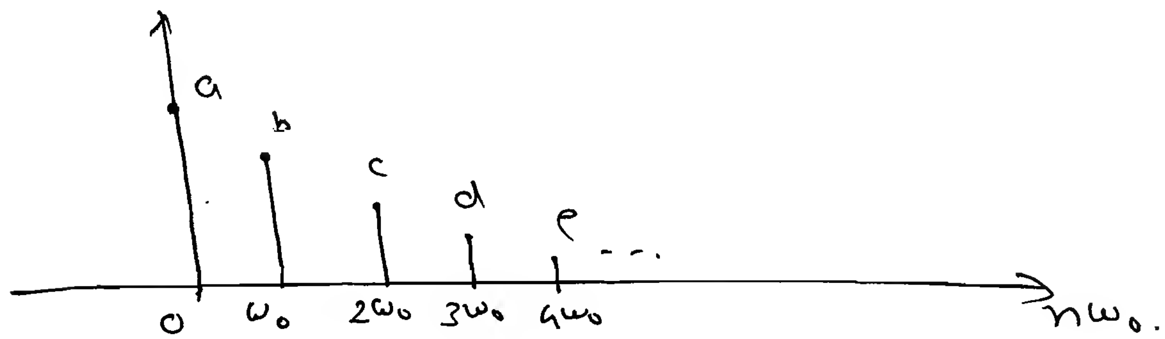
$$\therefore \quad x(\alpha t) = \sum_{n=-\infty}^{+\infty} C_n \cdot e^{-j\alpha\omega_0 n t}.$$

Now, $\boxed{\omega_0' = \alpha\omega_0.}$

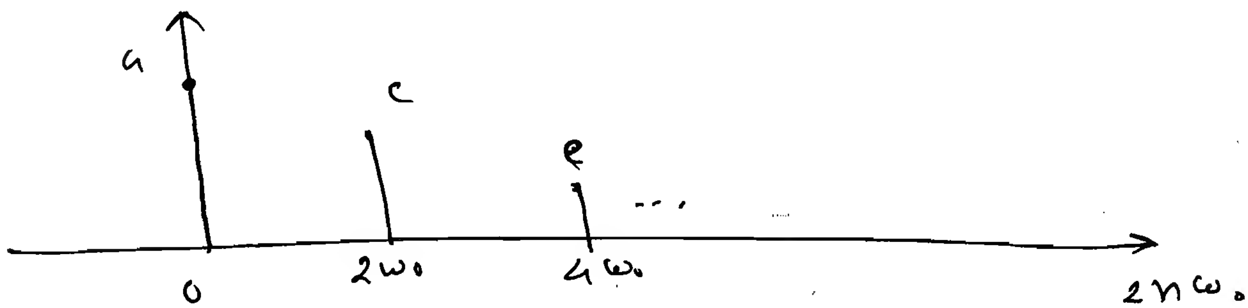
$$\frac{2\pi}{T'} = \alpha \times \frac{2\pi}{T}.$$

$$\therefore \boxed{T' = T/\alpha}$$

e.g. $x(t) \longrightarrow C_n|_{\omega_0}$



$\Rightarrow \quad x(2t) \longrightarrow C_n|_{2\omega_0}$



Note: Compression in time domain is expansion in freq. domain.

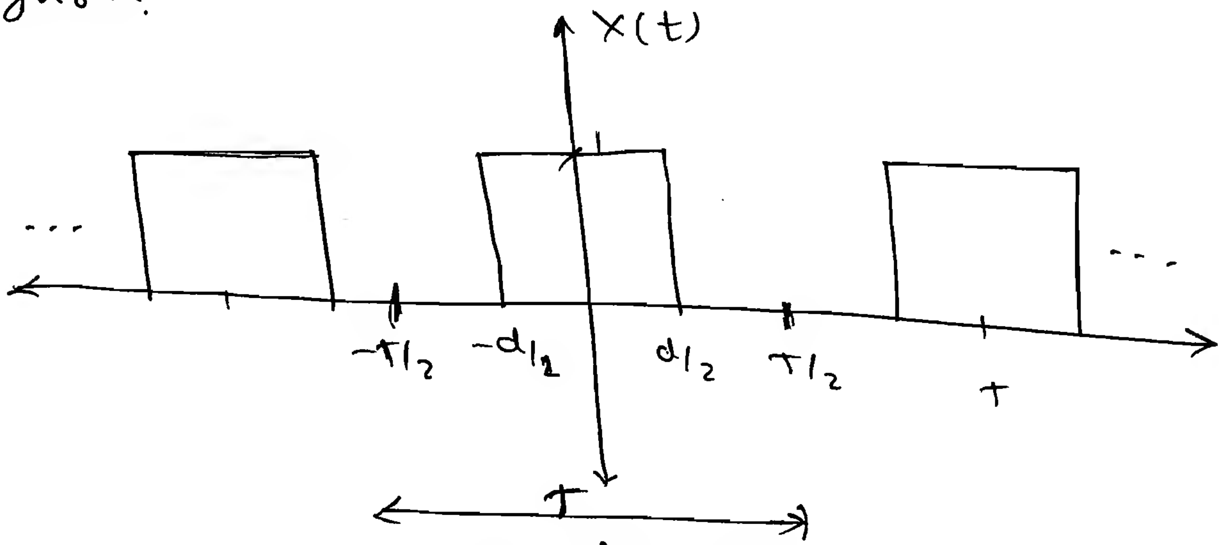
(5) Differentiation in time:

$\Rightarrow$ $x(t) \longleftrightarrow C_n$.

$\therefore$ $\dfrac{dx(t)}{dt} \longleftrightarrow (j\omega_0 n)\, C_n$:

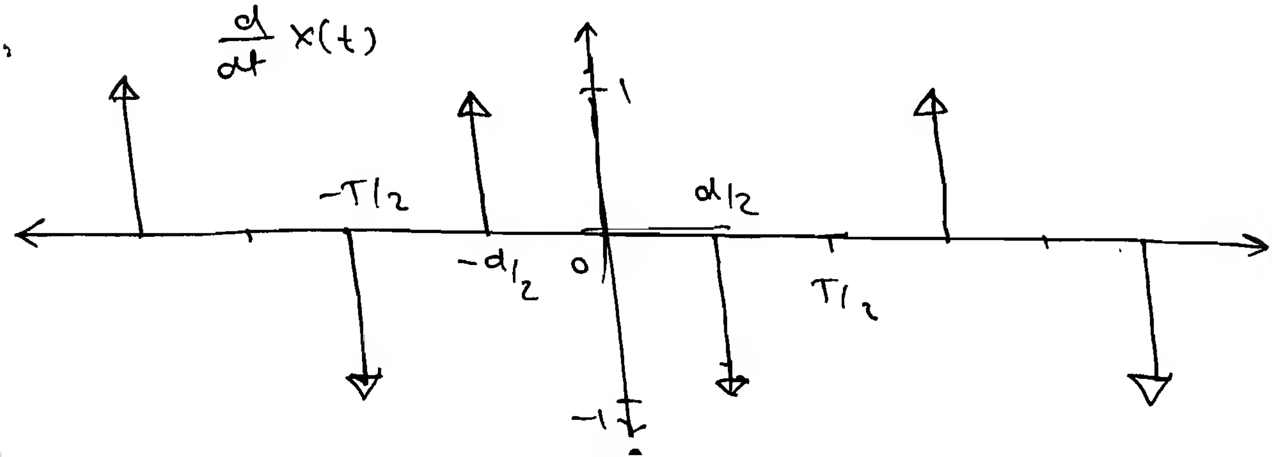
$\therefore$ $\dfrac{d^K x(t)}{dt^K} \longleftrightarrow (j\omega_0 n)^K . C_n$.

$\boxed{P\ 3.3.3}$ By using derivative method, find F.S. coefficient of the signal shown in figure?



$\omega_0 = \dfrac{2\pi}{T} = 2\pi/T$

Soln:



$\therefore$ ~~$d_{xxx/2}$~~

$$y(t) = \frac{d}{dt}\, x(t).$$

$$\therefore \quad d_n = (j\omega_0 n)\, c_n .$$

$$\therefore \quad c_n = \frac{d_n}{j\omega_0 n}.$$

$$\therefore \quad d_n = \frac{1}{T} \int_{0}^{T} y(t) \cdot e^{-j\omega_0 n t} \cdot dt .$$

$$\therefore \quad d_n = \frac{1}{T} \int_{0}^{T} \left[\delta(t + d/2) - \delta(t - d/2) \right] \cdot e^{-j\omega_0 n t} \cdot dt .$$

Now,

$$\int_{t_1}^{t_2} \delta(t - t_0) \cdot x(t)\, dt = x(t_0).$$

$$t_1 \le t_0 \le t_2 .$$

$$\therefore \quad d_n = \frac{1}{T} \left[e^{j\omega_0 n \frac{d}{2}} - e^{-j\omega_0 n d/2} \right]$$

$$\therefore \quad d_n = \frac{2j}{T} \left[\frac{e^{j\omega_0 n \frac{d}{2}} - e^{-j\omega_0 n d/2}}{2j} \right].$$

$$\therefore \quad d_n = \frac{2j}{T} \times \sin \omega_0 n \frac{d}{2} .$$

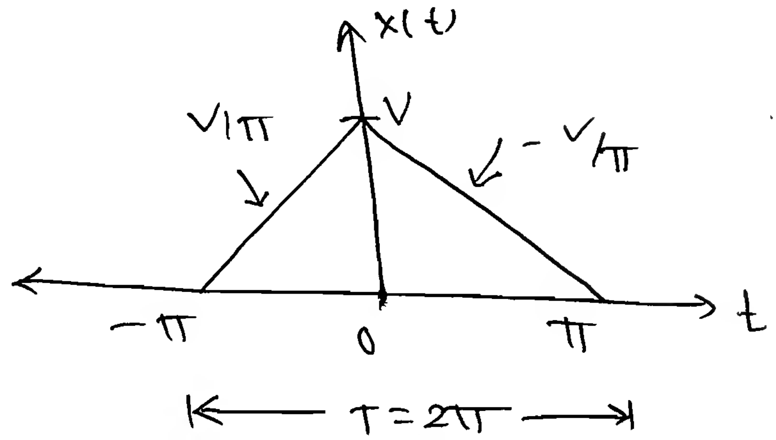
$$\therefore \quad \cancel{d_n} \quad C_n = \frac{d_n}{j\omega_0 n}.$$

$$\therefore \quad C_n = \frac{2j}{T \times j\omega_0 n} \times \sin \omega_0 n\, d/2$$

$$= \frac{2}{T \times \frac{2\pi}{T} \times n} \times \sin \omega_0 n \frac{d}{2}.$$

$$\boxed{C_n = \frac{\sin \omega_0 n \frac{d}{2}}{\pi n}}$$

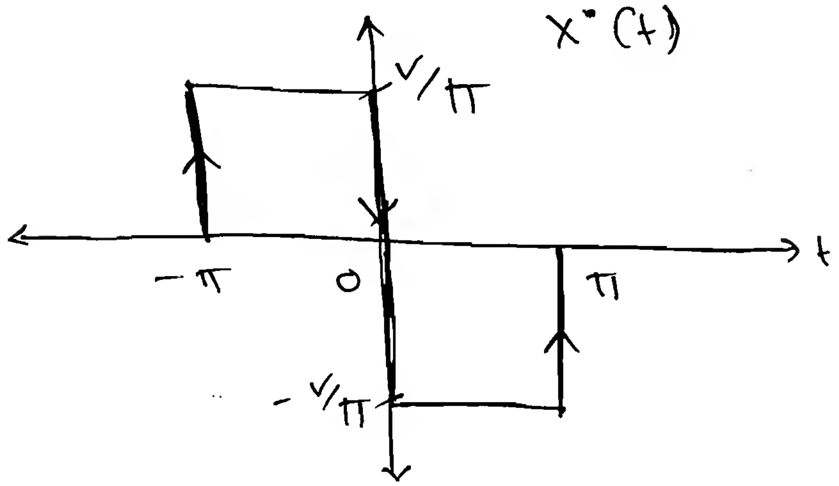
Q



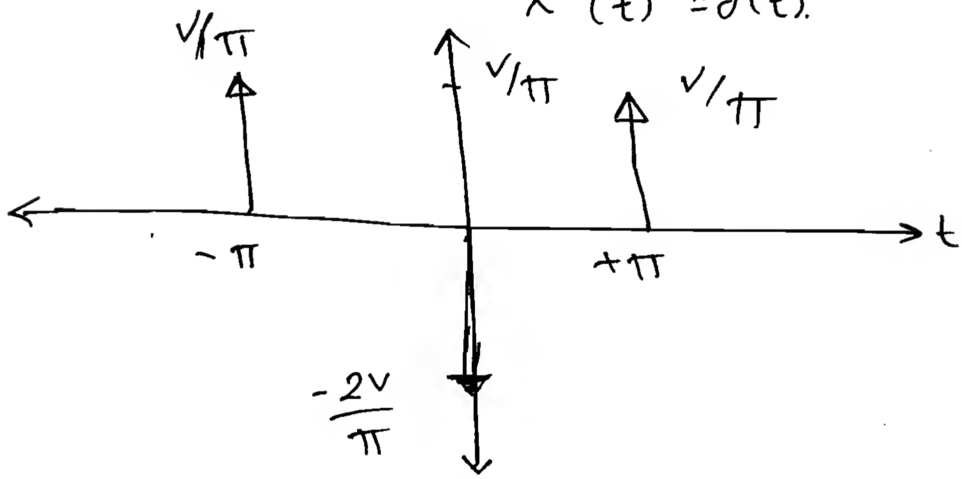
$\Rightarrow \omega_0 = 1$ rad/sec.

$\Downarrow$

$\Rightarrow$



$\Downarrow$ $x''(t) = y(t)$.



~~Sol~~: $\Rightarrow$ $y(t) = \dfrac{d^2 x(t)}{dt}$.

$\Downarrow$

$d_n = (j\omega_0 n)^2 \cdot C_n$.

$\therefore$ $C_n = \dfrac{d_n}{(j\omega_0 n)^2}$

$$\boxed{C_n = -\frac{d_n}{n^2}} \Rightarrow \boxed{C_n \propto |n^{-2}|.}$$

$(\because \omega_0 = 1 \text{ rad/sec})$.

$\Rightarrow$ $g(t) = \dfrac{dx(t)}{dt}$.

$g_n = (j\omega_0 n) \cdot C_n$.

$C_n = \dfrac{g_n}{(jn)}$

$\Rightarrow$ $\boxed{C_n \propto |n^{-1}|}$

$\boxed{\text{P 3.3.5}}$ A periodic signal has the F.S. representation $x(t) \xleftrightarrow[\omega_0 = \pi]{F.S.} C_n = -n \cdot 2^{-|n|}$. Without finding $x(t)$, find the F.S. representation [d_n & ω_0] if

(a) $y(t) = x(3t)$.

Soln:

$d_n = C_n \Big|_{\omega_0' = 3\omega_0}$.

$d_n = -n\, 2^{-|n|} \Big|_{\omega_0' = 3\omega_0}$.

(b) $y(t) = \frac{d}{dt} x(t)$.

Soln:

$$d_n = (j\omega_0 n) \cdot C_n \Big|_{\omega_0 = \pi}$$

$$\therefore \quad d_n = (j\pi n) \cdot (-n) \cdot 2^{-|n|}.$$

$$d_n = -j\pi n^2 \cdot 2^{-|n|} \Big|_{\omega_0 = \pi}.$$

(c) $y(t) = x(t-1)$.

Soln:

$$d_n = e^{-j\omega_0 n(1)} \cdot C_n.$$

$$= e^{-j\pi n} \cdot (-n) \cdot 2^{-|n|}.$$

$$= (-1)^n \cdot (-n) \cdot 2^{-|n|}.$$

$$d_n = n\,(-1)^{n+1} \cdot 2^{-|n|} \Big|_{\omega_0 = \pi}.$$

(d) $y(t) = \mathrm{Re}\{x(t)\}$.

Soln: $x(t) = x_R(t) + j\,x_I(t)$.

$$x^*(t) = x_R(t) - j\,x_I(t).$$

$$\therefore \quad x_R(t) = \frac{x(t) + x^*(t)}{2}$$

$$\therefore \quad x_I(t) = \frac{x(t) - x^*(t)}{2}$$

$$\boxed{\begin{aligned} \therefore x_R(t) &= \frac{x(t)+x^*(t)}{2} \xleftrightarrow{F.S.} \frac{C_n + C_{-n}^*}{2}. \\ x_I(t) &= \frac{x(t)-x^*(t)}{2} \xleftrightarrow{F.S.} \frac{C_n - C_{-n}^*}{2}. \end{aligned}}$$

(e) $y(t) = \cos 4\pi t \, x(t).$

Sol^n:

$$y(t) = \left[\frac{e^{j4\pi t} + e^{-j4\pi t}}{2}\right] x(t).$$

$$y(t) = \frac{\underbrace{e^{j4\pi t}}_{\leftarrow m=4} \cdot x(t) + \underbrace{e^{-j4\pi t}}_{\swarrow m=-4} \cdot x(t)}{2}.$$

$$\therefore d_n = \frac{C_{n-4} + C_{n+4}}{2}.$$

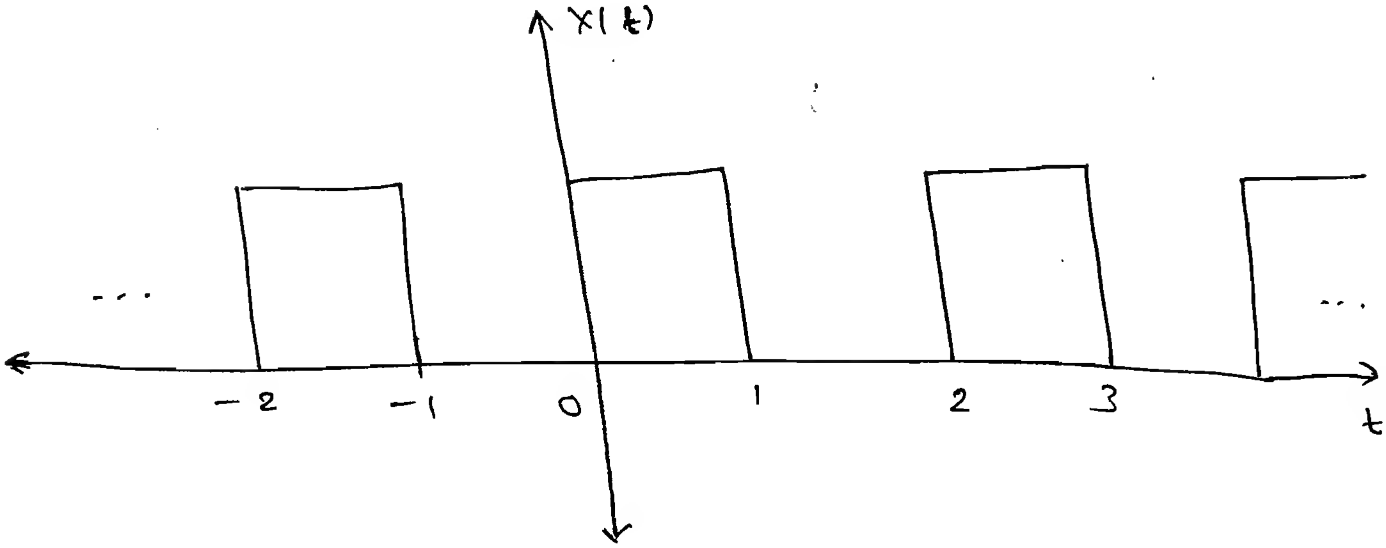
$$\therefore d_n = \frac{-(n-4) \cdot 2^{-|n-4|} + -(n+4) \cdot 2^{-|n+4|}}{2}.$$

6) <u>Parseval's</u> <u>Power</u> <u>theorem</u>:

$\Rightarrow$ The total average Power in Periodic signal is eaual to the Sum of the Squared amplitude of each harmonics.

$x(t) \longleftrightarrow C_n$ then $\boxed{\frac{1}{T}\int_0^T |x(t)|^2 \cdot dt = \sum_{n=-\infty}^{+\infty} |C_n|^2.}$

P 3.3.6. Find the Power up to II harmonic for the periodic signal shown in figure.



soln:

from Q : 3.2.12

$\Rightarrow$ Amp: 10 | $b_n = \frac{20}{n\pi}$ (odd n).

a_0: 5

a_n: 0

Here, Amp = 1 so,

$C_0 = a_0 = 1/2$.

$$C_n = \frac{a_n - jb_n}{2} = -\frac{j}{2}\left(\frac{2}{n\pi}\right) = -\frac{j}{n\pi} \quad \text{(odd n)}.$$

$\Rightarrow$ Power required upto II harmonic.

$$P = \sum_{n=-2}^{+2} |C_n|^2.$$

$$P = |C_{-2}|^2 + |C_{-1}|^2 + |C_0|^2 + |C_1|^2 + |C_2|^2$$

$$= \frac{1}{(2\pi)^2} + \frac{1}{(\pi)^2} + \left(\frac{1}{2}\right)^2 + \frac{1}{(\pi)^2} + \frac{1}{(2\pi)^2}.$$

$$P = 0.45 \text{ watts.}$$

Calculation of total power:

$$P = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

$$= \frac{1}{2}\int_0^1 (1)^2 . dt.$$

$$P = \frac{1}{2}$$

$$\boxed{P = 0.5 \text{ watts}}$$

Note: Maximum energy (or) power of any signal is always there only in the low freq. region.

$\boxed{P\ 3.3.7}$ The F.S. coefficients, of a periodic signal $x(t)$ is expressed as $x(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_0 t}$ are given by

$C_{-2} = 2 - j1$; $C_{-1} = 0.5 + j0.2$; $C_0 = j2$; $C_1 = 0.5 - j0.2$; $C_2 = 2 + j1$; $C_n = 0$ for $|n| > 2$.

Which of the following is TRUE ?

(a) $x(t)$ has finite energy because only finitely many coefficients are non zero.

(b) $x(t)$ has zero average value because it is periodic.

(c) the imaginary part of $x(t)$ it is constant.

(d) the real part of $x(t)$ is even.

Soln: → Every periodic signal is power signal. and it energy is ∞.

so option A is wrong.

→ As $C_0 = j2$ is given i.e $x(t)$ has non zero average value and it and it indicates it is complex. hence option b is also wrong.

→ $x(t) = x_R(t) + x_I(t)$.

$$\therefore \quad C_n = \frac{1}{T} \int_{-\infty}^{+\infty} x(t) . e^{-j\omega_0 n t} \, dt .$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} [\underbrace{x_R(t)}_{0} + x_I(t)] . e^{-j\omega_0 n t} \, dt .$$

$$\therefore \quad C_0 = 0 + j2 \text{ (constant)}$$

So, real part of $x(t)$ is 0.

hence, the imaginary part of $x(t)$ is constant.

So, Ans → (c).

* System with Periodic Inputs:

$\Rightarrow$ i/p $x(t) = e^{j\omega t} \xrightarrow{L.T.I.}$ o/p $y(t) = e^{j\omega t} \cdot H(\omega)$.

$\rightarrow$ $$y(t) = \int_{-\infty}^{+\infty} x(t-\tau) \cdot h(\tau)\, d\tau.$$

$$= \int_{-\infty}^{+\infty} e^{j\omega(t-\tau)} \cdot h(\tau) d\tau.$$

$$= e^{j\omega t} \cdot \int_{-\infty}^{+\infty} e^{-j\omega\tau} \cdot h(\tau) \cdot d\tau.$$

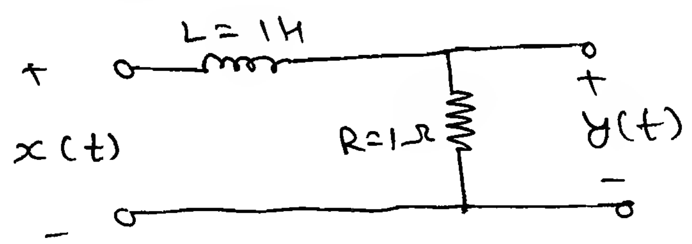
$$\boxed{y(t) = e^{j\omega t} \cdot H(\omega).}$$

$\Rightarrow$ $$x(t) = \sum_{n=-\infty}^{+\infty} C_n \cdot e^{jn\omega_0 t}.$$

$\downarrow$

$$y(t) = \sum_{n=-\infty}^{+\infty} C_n \cdot H(n\omega_0) \cdot e^{jn\omega_0 t}.$$

P3.4.1 Find the output voltage of the system shown in figure, if the input voltage is $x(t) = 4\cos 2t$



$\therefore \quad \frac{dy(t)}{dt} + y(t) = x(t).$

$\therefore \quad j\omega e^{j\omega t} H(\omega) + e^{j\omega t} \cdot H(\omega) = e^{j\omega t}.$

$\therefore \quad H(\omega) = \frac{1}{1 + j\omega}.$

$\therefore \quad H(n\omega_0) = \frac{1}{1 + j\omega_0 n} = \frac{1}{1 + j2n}.$

$(\because \omega_0 = 2).$

$\Rightarrow \quad x(t) = 4\cos 2t$

$= 2e^{+j(1)(2)t} + 2e^{j(-1)(2)t}.$

$\therefore \quad y(t) = 2 \cdot e^{j(1)(2)t} \cdot H(1) + 2e^{-j(2)t} \cdot H(-1).$

$\therefore \quad y(t) = 2\left[\frac{1}{1+j2}\right] e^{j2t} + 2\left[\frac{1}{1-j2}\right] e^{-j2t}.$

$= K\cos(2t + \phi).$

Note:

Signal Analysis] F.S. & F.T.

System Analysis] L.T & Z.T.